

General Linear Model III

consider

$$Y_t = \beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + U_t$$

$$\hat{\beta}_1 = \frac{\sum \hat{u}_{t1} y_t}{\sum \hat{u}_{t1}^2}$$

where u_{ti} : the residuals from the regression of the i th explanatory variable on the other

Similarly,

$$\text{from: } Y_t = \beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + \dots + \beta_k X_{tk} + U_t$$

$$\hat{\beta}_i = \frac{\sum \hat{u}_{ti} y_t}{\sum \hat{u}_{ti}^2}$$

where, $\hat{u}_{ti}$ is from:

$$X_{ti} = C_{i0} + C_{i1} X_{t1} + \dots + C_{i,i-1} X_{t,i-1} + C_{i,i+1} X_{t,i+1} + \dots + U_{ti}$$

$$R^2 = \frac{\sum \hat{y}_t^2}{\sum y_t^2}$$

$$R^2 = 1 - \frac{\sum \hat{u}_t^2}{\sum y_t^2}$$

$$R^2 = \frac{\sum \hat{y}_t^2}{\sum y_t^2}$$

Sampling Properties

$$Y_t = \beta_0 + \sum_{i=1}^k \beta_i X_{ti} + U_t$$

$$\hat{\beta}_i = \frac{\sum \hat{u}_{ti} y_t}{\sum \hat{u}_{ti}^2}, \quad i \geq 1, \dots, k$$

$$\text{define } a_{ti} = \frac{\hat{u}_{ti}}{\sum \hat{u}_{ti}^2}$$

$$\begin{aligned} \Rightarrow \hat{\beta}_i &= \sum a_{ti} y_t = \sum a_{ti} (y_t - \bar{y}) \\ &= \sum a_{ti} y_t - \underbrace{\bar{y} \sum a_{ti}}_{=0 \text{ since } \sum a_{ti} = 0} \end{aligned}$$

$$\hat{\beta}_i = \sum a_{ti} y_t$$

↳ linear

$$\begin{aligned} &= \sum a_{ti} (\beta_0 + \beta_1 x_{t1} + \dots + \beta_k x_{tk} + u_t) \\ &= \beta_0 \sum a_{ti} + \beta_1 \sum a_{ti} x_{t1} + \dots + \beta_k \sum a_{ti} x_{tk} + \sum a_{ti} u_t \\ \sum \hat{u}_{ti} x_{tj} &= 0 \text{ The correlation between } \hat{u}_{ti} \text{ and } x_{tj} \text{ is 0. } \sum a_{ti} x_{tj} = 0 \quad i \neq j \\ \hat{\beta}_i &= \beta_i + \sum a_{ti} u_t \end{aligned}$$

Estimated Variance

$$\hat{\beta}_i = \beta_i + \sum a_{ti} u_t$$

$$E(\hat{\beta}_i) = \beta_i \text{ since } E(u_t) = 0$$

$$\Rightarrow \hat{\beta}_i : \text{unbiased}$$

$$\begin{aligned} \text{Var}(\hat{\beta}_i) &= E(\hat{\beta}_i - E(\hat{\beta}_i))^2 = E(\hat{\beta}_i - \beta_i)^2 \\ &= E\left[\sum a_{ti} u_t\right]^2 \end{aligned}$$

$$\text{Var}(\hat{\beta}_i) = \sigma_{\beta_i}^2 = \frac{\sigma^2}{\sum \hat{u}_{ti}^2}, \quad i = 1 \dots k$$

$$\text{Est. Var.}(\hat{\beta}_i) = \hat{\sigma}_{\beta_i}^2 = \frac{\hat{\sigma}^2}{\sum \hat{u}_{ti}^2}$$

Correlation Coefficient

$$Y_t = \beta_0 + \sum_{i=1}^k \beta_i X_{ti} + u_t$$

r_{y, x_i} : simple correlation coefficient

$$r_{y, x_i} = \frac{\sum x_{ti} y_t}{\sqrt{\sum x_{ti}^2} \sqrt{\sum y_t^2}}$$

r_{y, x_i} may not be an appropriate if x_i contains also the effect of the other variable

for example: $Y_t = \beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + u_t$



therefore,

we need a measure which shows the Net degree of linear association between Y and X_i

The Partial Correlation Coefficient

$r_{y, i}$: Partial correlation coefficient, net on the influence of the remainder exp variables

consider

- i) Regress Y_t on all of the explained variable except x_i
 $\hookrightarrow \hat{u}_t, y_i$
- ii) Regress X_i on all of the remaining explanatory variables
 $\hookrightarrow \hat{v}_{ti}$

$$1 - Y_t = a_0 + a_1 X_{t,1} + \dots + a_{i-1} X_{t,i-1} + a_{i+1} X_{t,i+1} + \dots + a_{k+1} X_{t,k+1}$$

$$2 - X_{t,i} = b_0 + b_1 X_{t,1} + \dots + b_{i-1} X_{t,i-1} + b_{i+1} X_{t,i+1} + \dots + b_{k+1} X_{t,k+1}$$

here:

$\hat{u}_{t,y_i}$: the part of Y_t which is not explained by the remaining explanatory variables

$\hat{u}_{t,i}$: The part of $X_{t,i}$ which is not explained by the remaining expl. variables

Note that

$$t_{\hat{\beta}_i}^2 = \frac{r_{y,i}^2}{1 - r_{y,i}^2} \cdot \frac{T - k - 1}{1}$$

$$r_{y,i}^2 = \frac{t_{\hat{\beta}_i}^2}{t_{\hat{\beta}_i}^2 + \underbrace{df}_{T - k - 1}}$$

where, $t_{\hat{\beta}_i} = \frac{\hat{\beta}_i}{\hat{\sigma}_{\hat{\beta}_i}}$ to test $H_0: \hat{\beta}_i = 0$

The following consumption func is estimated by using 20 observations

$$C_t = 400 + 0.8 Y_t$$

(40) (0.2)

$$R^2 = 0.900, SSR = 1200$$

$$C = Y - S$$

$$(1) S_t = \gamma_0 + \gamma_1 Y_t + u_{t1}$$

$$(2) S_t = \beta_0 + \beta_1 Y_t + \beta_2 C_t + u_{t2}$$

a) for 1 find y_1, R^2, SSR

$$S_e = Y_t - (400 + 0.8Y_t) = Y_t - 400 - 0.8Y_t = 0.2Y_t - 400$$

$$y_0 = -400 \quad y_1 = 0.2$$

$$SSR = 1200 \quad R^2 = 0.900$$

b) for 2 $\beta_1, R^2, \beta_2, SSR$

$$C_t = Y_t - S_t$$

$$S_t = Y_t - C_t$$

perfect model with no estimation error

$$S_t = 1 \cdot Y_t - 1 \cdot C_t + 0$$

$$\beta_1 = 1 \quad \beta_2 = -1 \quad \beta_0 = 0 \quad R^2 = 1.0 \quad SSR = 0$$