

Problem Set III

Section I: Chapter 5

Question 1

Consider a one-period economy that consists of many households that are identical with the following preferences, where c is the consumption, l is leisure and γ is a preference shock:

$$U(c, l) = \log(c) + \gamma \log(l)$$

The households have a limited amount of time at their disposal, h , and have the choice to use it doing leisurely activities that give them utility (going out, reading a book etc.) or work (let the labor supply of each household be N^s) and get paid the market wage w . Each household also owns an equal share of the firms in this economy and get profit income π from them. There is also a government that uses lump-sum taxation to extract T from the households, to finance its expenditures G . Production of the consumption good in this economy is done by a large number of identical firms that have the following technology, where K is the fixed capital stock of the economy and N^d is the labor demand by the firm and z is the exogenous TFP shock.

$$Y = zf(K, N^d) = zK^\alpha N^{1-\alpha}$$

Households and firms interact in the labor market exchanging labor in return for wages (note that this is a real-economy and wages are paid in terms of consumption good, i.e., $w = 2$ means one hour of labor will get you 2 units of consumption good).

- What are the exogenous variables in this model? What are the endogenous variables? What are the parameters of the model?
- Define the competitive equilibrium of this economy; be precise as in your textbook.
- Write down the budget and time constraint of the household (you can assume the total number of hours available is normalized to 1, i.e., $h = 1$). Write down the utility maximization problem of the representative household. What are the choice variables for the household?
- Write down the profit maximization problem of the representative firm, what does the firm choose?

- e. Solve the problem of the household to get optimal consumption level, c^* , and labor supply N^{s*} for a given level of wage, w (i.e. the consumption and labor supply functions you find will only be functions of the exogenous variables, parameters and w).
- f. Solve the problem of the firm to get the labor demand, N^{d*} .
- g. What is the equilibrium level of w that will clear the labor market, i.e. $N^s = N^d$? What is the equilibrium level of employment, N^* (and leisure, l^*)? Equilibrium level of output, Y^* , and aggregate consumption C^* ? (The starred equilibrium variables will only be functions of exogenous variables and parameters of the model).
- h. Using the government budget $G - T$, and budget constraint of the representative household, show that when the labor market is in equilibrium, i.e. $N^s = N^d = N^*$, the aggregate resource constraint holds, i.e. $Y^* = C^* + G$. Now consider this country comes under the rule of a benevolent dictator. This dictator can dictate inputs to production by the representative firm, can force the consumer to supply the appropriate quantity of labor, and then distributes consumption goods and leisure hours to the consumer; all in a way that makes the consumer as well off as possible. There is no need for the existence of decentralized markets and prices for allocation mechanism as the dictator claims it can do a better job. The problem of the dictator is to maximize the utility of the household subject to the production technology, $Y = zf(K, N)$ and the aggregate resource constraint $Y = C + G$.
- i. Write down the maximization of social welfare problem of the benevolent dictator.
- j. Solve this problem to get the socially optimal level of consumption, leisure, employment and output. Is this allocation Pareto optimal?
- k. Is the dictator's claim of being able to do a better job than markets supported? If you were an economic advisor to this dictator, which economic theorem would you have cited to convince him his claim would not hold?

Question 2

Consider the following economy. A government finances its exogenous spending g through a labor income tax. In particular, the government spending g is equal to the amount of tax collected T . Hence, $g = T$. A representative firm produces goods using a linear technology:

$$y = zn$$

where y is the output, z is a constant exogenous technology, and n is the labor input. A representative consumer solves:

$$\begin{aligned} \max_{c, l} \quad & u(c, l) \\ \text{s.t.} \quad & c = (1 - \tau_w)w(1 - l) \end{aligned} \tag{1}$$

Hence, the tax imposed by the government is a fraction τ_w of the wage-bill $w(1 - l)$ of the household. The government collects only labor taxes, i.e.,

$$T = \tau_w w(1 - l)$$

- Find the optimal demand schedule for labor for the firm.
- Find the optimal supply schedule of labor for the households.
- Define accurately a competitive equilibrium for this economy.
- Write the equivalent social planner problem.
- Find the equilibrium that yields the optimal level of labor in the economy.
- Are the competitive equilibrium and the solution to the social planner problem equivalent? Explain.
- Use a graph in the space c (y-axis) and l (x-axis). Display the social planner equilibrium and the competitive equilibrium in the same graph. Explain carefully all the curves you plot.

Question 3

Suppose that the substitution effect of an increase in the real wage is always larger than the income effect for the representative consumer. Also assume that the economy is always in the low tax- rate equilibrium on the good side of the Laffer curve. Determine the effects of an increase in total factor productivity, z , on the Laffer curve, on the equilibrium tax rate, and on consumption, leisure, the quantity of labor supplied, and output.

Question 4

In the simplified model proportional taxation there can be two equilibria, one with a high tax rate and one with a low tax rate. Now, suppose that government spending increases determine the effects of an increase in G on consumption, leisure, labor supply, real output and the tax rate in a high tax rate equilibrium and in a low tax rate equilibrium. How do your results differ? Explain why

Question 5

Suppose that total factor productivity, z , affects the productivity of government production just as it affects private production. That is, suppose that when the government collects taxes, it acquires goods that are then turned into government-produced goods according to $G = zT$ so that z units of government goods are produced for each unit of taxes collected. With the government setting G , an increase in z implies that smaller quantity of taxes are required to finance the given quantity of government purchases G . Under these circumstances, using a diagram determine the effects of an increase in z on output, consumption, employment, and the real wage, treating G as given. Explain your results.

Question 6

Consider the model of public goods in the last section of this chapter.

- Suppose that preferences over private consumption C and public goods G are such that these two goods are perfect substitutes; that is, the marginal rate of substitution of public goods for private goods is a constant $b > 0$. Determine the optimal quantity of public goods that the government should provide, and interpret your results. Make sure you show all of the relevant cases. What happens when b changes, or when q changes?
- Repeat part (a), except with perfect complements preferences; that is, for the case where the representative consumer always wishes to consume private consumption goods and public goods in fixed proportions, or $C = aG$, with $a > 0$.

Section II: Chapter 9

Question 7

Suppose that there is a household who lives for two periods. Its lifetime utility function is:

$$U = u(C_t) + \beta u(C_{t+1})$$

$u(\cdot)$ is a function that is increasing and concave (the first derivative is **positive** and the second derivative is **negative**). The household faces two within-period budget constraints:

$$C_t + S_t = Y_t$$

$$C_{t+1} = Y_{t+1} + (1 + r_t)S_t$$

r_t is the interest rate on the stock of savings brought from period t into period $t + 1$. The household would like to pick consumption and saving to maximize lifetime utility subject to the two budget constraints.

- a. Solve for S_t from either the t or $t + 1$ budget constraint, plug it into the other, and derive the unified, intertemporal budget constraint. In words, describe what the intertemporal budget constraint says.
- b. Solve for C_{t+1} in terms of C_t , Y_t , Y_{t+1} , and r_t in the intertemporal budget constraint you derived in part (a). Plug this into the objective function, which turns the problem into an unconstrained maximization problem just over C_t . Derive the Euler equation (first-order necessary condition for a maximum). Provide an economic interpretation of the Euler equation.
- c. Define an indifference curve and derive an expression for the slope of the indifference curve.
- d. Combine the indifference curve with a graphical representation of the intertemporal budget constraint (what we call in the graph the “budget line”). Graphically show the optimal consumption bundle.
- e. Suppose that there is an increase in Y_t . Graphically show how this ought to affect C_t and C_{t+1} .
- f. Suppose that there is an increase in Y_{t+1} . Graphically show how this ought to affect C_t and C_{t+1} .
- g. How does first-period saving, S_t , react differently to an increase in Y_t relative to an increase in Y_{t+1} ?

Question 8

Assume a consumer who has current period income $y=200$, future period income $y'=150$, current and future taxes $t=40$ and $t'=50$, respectively, and faces a market real interest rate of $r=0.05$, or 5% per period. The consumer would like to consume equal amounts in both periods, that is, he or she would like to set $c=c'$ if possible. However, this consumer is faced with a credit market imperfection, in that he or she cannot borrow at all, that is $s \geq 0$.

- a. Show the consumer’s lifetime budget constraint and indifference curves in a diagram.
- b. Calculate his or her optimal current period and future period consumption and optimal saving, and show this in your diagram.
- c. Suppose that everything remains unchanged, except that now $t = 20$ and $t' = 71$. Calculate the effects on current and future consumption and optimal saving, and show this in your diagram.
- d. Now, suppose alternatively that $y = 100$. Repeat parts (a) to (c), and explain any differences.