

Statistics for Economists II

Lecture 8: More Confidence Intervals

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Spring 2025–2026

Roadmap for Today

- 1 CI for Population Variance
- 2 CI for Population Proportion
- 3 CI for Two Populations
- 4 Finite Population Correction
- 5 Summary: Common Mistakes to Avoid
- 6 Review Problems

Why Care About Variance?

- In many economic contexts, the **spread** matters as much as the center

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- **Income inequality:** high variance in wages $\Rightarrow$ greater inequality
- We already know how to build CIs for μ .
Today: CIs for σ^2

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The Pivotal Quantity for σ^2

Key Result (from Week 2)

If $X_1, \dots, X_n \stackrel{\text{i.i.d.}}{\sim} N(\mu, \sigma^2)$, then:

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

where $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$ is the sample variance.

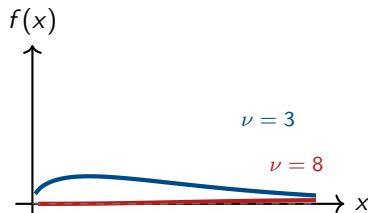
This result is the **key** to building a CI for σ^2 . Just as we used $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$ to build CIs for μ , we use the chi-squared pivot for σ^2 .

Important: Unlike CIs for the mean (which work approximately via the CLT for any population), CIs for variance **require normality**. They are sensitive to departures from this assumption.

The Chi-Squared Distribution: A Reminder

Properties of $\chi^2(\nu)$:

- Defined for $x \geq 0$ only (variance is non-negative!)
- $\mathbb{E}[\chi^2(\nu)] = \nu$
- $\text{Var}[\chi^2(\nu)] = 2\nu$
- **Right-skewed**, especially for small ν
- Becomes more symmetric as ν grows



Critical Difference from Normal-Based CIs

Because χ^2 is **asymmetric**, the CI for σ^2 will **not** be symmetric around S^2 . We need *two* different critical values: $\chi^2_{\alpha/2}$ and $\chi^2_{1-\alpha/2}$.

Constructing the CI for σ^2

We want to find a and b such that:

$$\mathbb{P}\left(a \leq \frac{(n-1)S^2}{\sigma^2} \leq b\right) = 1 - \alpha$$

Choose $a = \chi_{1-\alpha/2, n-1}^2$ and $b = \chi_{\alpha/2, n-1}^2$. Inverting:

$(1 - \alpha) \times 100\%$ CI for σ^2

$$\left(\frac{(n-1)S^2}{\chi_{\alpha/2, n-1}^2}, \frac{(n-1)S^2}{\chi_{1-\alpha/2, n-1}^2} \right)$$

Notice the **inversion**: the larger chi-squared value ($\chi_{\alpha/2}^2$) goes in the *denominator* of the **lower** bound, making the lower bound smaller.

For a CI for σ (the standard deviation), simply take square roots of both endpoints.

Example: Quality Control in Manufacturing

Problem: A factory produces bolts with a target diameter. A sample of $n = 20$ bolts yields a sample variance of $S^2 = 0.045 \text{ mm}^2$. Find a 95% CI for the population variance σ^2 .

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Solution:

- Degrees of freedom: $\nu = n - 1 = 19$
- Critical values from $\chi^2(19)$ table:

$$\chi_{0.025, 19}^2 = 32.852 \quad \chi_{0.975, 19}^2 = 8.907$$

- Lower bound: $\frac{19 \times 0.045}{32.852} = \frac{0.855}{32.852} = 0.0260$
- Upper bound: $\frac{19 \times 0.045}{8.907} = \frac{0.855}{8.907} = 0.0960$

$$\boxed{95\% \text{ CI for } \sigma^2 : (0.0260, 0.0960) \text{ mm}^2}$$

The interval is **not symmetric** around $S^2 = 0.045$: it extends further to the right (0.051 above) than to the left (0.019 below). This reflects the right skewness of the χ^2 distribution.

Confidence Interval for a Proportion: Review & Extension

Setup: $X_1, \dots, X_n \stackrel{\text{i.i.d.}}{\sim} \text{Bernoulli}(p)$

Estimator: $\hat{p} = \bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$

By CLT (for large n):

$$\frac{\hat{p} - p}{\sqrt{p(1-p)/n}} \stackrel{\text{approx}}{\sim} N(0, 1)$$

$(1 - \alpha) \times 100\%$ CI for p

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

Rule of thumb: the normal approximation works well when $n\hat{p} \geq 5$ and $n(1 - \hat{p}) \geq 5$.

We replaced the unknown p in the standard error with $\hat{p}$. This is valid for large n by Slutsky's theorem (the estimation error in $\hat{p}$ vanishes). This is the **Wald interval**—simple but can have poor coverage for extreme p or small n .

Sample Size Determination for Proportions

Question: How large should n be to achieve a desired margin of error E ?

The margin of error is $E = z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$. Solving for n :

Required Sample Size

$$n = \left(\frac{z_{\alpha/2}}{E} \right)^2 \hat{p}(1 - \hat{p})$$

Conservative bound (if no prior estimate of p): use $\hat{p} = 0.5$, which maximizes $\hat{p}(1 - \hat{p})$:

$$n = \left(\frac{z_{\alpha/2}}{E} \right)^2 \times 0.25$$

Why $p = 0.5$? Because $p(1 - p)$ is maximized at $p = 0.5$. Using this gives the **largest** (most conservative) sample size, guaranteeing the desired precision *regardless* of the true p . In practice, if you have a rough prior guess, use it to get a smaller n .

Example: Election Polling

Problem: A polling firm wants to estimate the proportion of voters supporting a candidate. They want the margin of error to be at most 3 percentage points at the 95% confidence level. How many voters should they survey?

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$$n = \left(\frac{1.96}{0.03} \right)^2 \times 0.25 = (65.33)^2 \times 0.25 = 4268.44 \times 0.25 = 1067.1$$

$$n \geq 1,068 \text{ voters}$$

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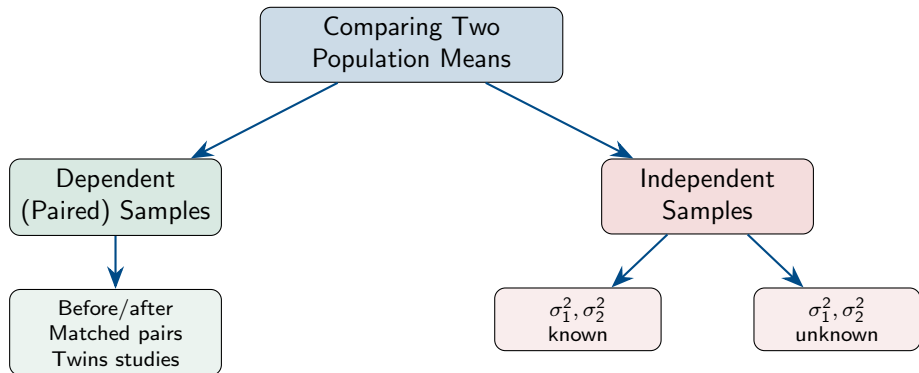
$$n \geq 1,068 \text{ voters}$$

If a prior poll suggested $\hat{p} \approx 0.40$:

$$n = \left(\frac{1.96}{0.03} \right)^2 \times 0.40 \times 0.60 = 4268.44 \times 0.24 = 1024.4 \implies n \geq 1,025$$

Using prior information saved about 40 respondents. In large-scale surveys, even small reductions in n save significant time and money. This is why pollsters run pilot surveys.

Comparing Two Populations: The Big Picture



The key question is whether the two samples are **linked observation-by-observation** (paired) or **drawn independently** from two populations. The statistical approach differs fundamentally.

Paired (Dependent) Samples

Setup: We observe pairs $(X_1, Y_1), (X_2, Y_2), \dots, (X_n, Y_n)$.

Key idea: Define the **differences** $D_i = X_i - Y_i$. Then:

- $D_1, D_2, \dots, D_n$ form a *single* random sample
- The problem **reduces** to a one-sample CI for $\mu_D = \mu_X - \mu_Y$

$(1 - \alpha) \times 100\%$ CI for $\mu_D = \mu_X - \mu_Y$

$$\bar{D} \pm t_{\alpha/2, n-1} \cdot \frac{S_D}{\sqrt{n}}$$

where $\bar{D} = \frac{1}{n} \sum D_i$ and $S_D^2 = \frac{1}{n-1} \sum (D_i - \bar{D})^2$.

By taking differences, we **eliminate** variability between subjects and isolate the **treatment effect**. This is why paired designs are more powerful—they “control for” individual heterogeneity.

Example: Treatment Effect (Paired Design)

Problem: An economist studies the effect of a job training program. Monthly wages (in 1000 TL) for 8 workers are measured *before* and *after* the program:

Worker	1	2	3	4	5	6	7	8
Before (X_i)	3.2	4.1	2.8	5.0	3.5	4.3	2.9	3.8
After (Y_i)	3.8	4.5	3.4	5.2	4.1	4.6	3.5	4.3
$D_i = Y_i - X_i$	0.6	0.4	0.6	0.2	0.6	0.3	0.6	0.5

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$D_i = Y_i - X_i$	0.6	0.4	0.6	0.2	0.6	0.3	0.6	0.5

Solution: $\bar{D} = \frac{0.6+0.4+0.6+0.2+0.6+0.3+0.6+0.5}{8} = \frac{3.8}{8} = 0.475$

$$S_D = \sqrt{\frac{\sum (D_i - 0.475)^2}{7}} = \sqrt{\frac{0.1488}{7}} \approx 0.1458$$

$$t_{0.025, 7} = 2.365$$

$$95\% \text{ CI : } 0.475 \pm 2.365 \times \frac{0.1458}{\sqrt{8}} = 0.475 \pm 0.122 = \boxed{(0.353, 0.597)}$$

Interpreting the Paired CI Result

Interpretation

We are 95% confident that the average wage increase due to the training program is between **353 TL** and **597 TL** per month.

Since the entire CI lies **above zero**, we can conclude (at the 95% level) that the training program has a **positive effect** on wages.

Why paired? Workers have vastly different baseline wages (2,800 to 5,000 TL). If we had used an independent samples approach, this between-worker variability would have *inflated* our standard error and made it harder to detect the effect. The paired design controls for individual differences.

Common paired scenarios in economics:

- Before/after policy interventions
- Same firms observed in two periods
- Matched control-treatment designs

Independent Samples: Known Variances

Setup: Two **independent** samples:

- Sample 1: $X_1, \dots, X_{n_1} \stackrel{\text{i.i.d.}}{\sim} N(\mu_1, \sigma_1^2)$, with σ_1^2 known
- Sample 2: $Y_1, \dots, Y_{n_2} \stackrel{\text{i.i.d.}}{\sim} N(\mu_2, \sigma_2^2)$, with σ_2^2 known

Then $(\bar{X} - \bar{Y}) \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$

$(1 - \alpha) \times 100\%$ CI for $\mu_1 - \mu_2$ (known variances)

$$(\bar{X} - \bar{Y}) \pm z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

This is the simplest case. In practice, known variances are rare—but this formula appears in large-sample problems where S_1^2 and S_2^2 can substitute reliably for σ_1^2 and σ_2^2 .

Independent Samples: Unknown but Equal Variances

If $\sigma_1^2 = \sigma_2^2 = \sigma^2$ (unknown), we **pool** the two sample variances:

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

$(1 - \alpha) \times 100\%$ CI for $\mu_1 - \mu_2$ (pooled)

$$(\bar{X} - \bar{Y}) \pm t_{\alpha/2, n_1+n_2-2} \cdot S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Pooling combines the information from both samples to get a **better estimate** of the common variance. The pooled estimator S_p^2 is a weighted average of S_1^2 and S_2^2 , with weights proportional to degrees of freedom.

When to pool: Only when you believe (or can test) that the two populations have equal variances. A rough rule: if $\max(S_1^2, S_2^2)/\min(S_1^2, S_2^2) < 2$, pooling is reasonable.

Independent Samples: Unknown and Unequal Variances

When $\sigma_1^2 \neq \sigma_2^2$ and both are unknown, use the **Welch (Satterthwaite)** approximation:

Welch's CI for $\mu_1 - \mu_2$

$$(\bar{X} - \bar{Y}) \pm t_{\alpha/2, \nu^*} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

with approximate degrees of freedom:

$$\nu^* = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{(S_1^2/n_1)^2}{n_1-1} + \frac{(S_2^2/n_2)^2}{n_2-1}}$$

(round ν^* down to the nearest integer)

The Welch formula is the **safer default** in practice—it works whether or not variances are equal. The Satterthwaite formula for ν^* looks intimidating, but it simply adjusts the degrees of freedom to account for the unequal variances.

Example: Comparing Wages Between Two Regions

Problem: An economist compares monthly wages (1000 TL) in two regions:

- Region A: $n_1 = 25$, $\bar{X}_1 = 8.4$, $S_1^2 = 2.56$
- Region B: $n_2 = 30$, $\bar{X}_2 = 7.1$, $S_2^2 = 3.24$

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Solution:

$$\bar{X}_1 - \bar{X}_2 = 8.4 - 7.1 = 1.3$$

$$SE = \sqrt{\frac{2.56}{25} + \frac{3.24}{30}} = \sqrt{0.1024 + 0.1080} = \sqrt{0.2104} = 0.4587$$

$$\nu^* = \frac{(0.2104)^2}{\frac{(0.1024)^2}{24} + \frac{(0.1080)^2}{29}} = \frac{0.04427}{0.000437 + 0.000402} = \frac{0.04427}{0.000839} \approx 52.8 \implies \nu^* = 52$$

$$t_{0.025, 52} \approx 2.007$$

$$95\% \text{ CI : } 1.3 \pm 2.007 \times 0.4587 = 1.3 \pm 0.920 = \boxed{(0.380, 2.220)}$$

Interpreting the Two-Sample CI

Interpretation

We are 95% confident that workers in Region A earn between **380 TL and 2,220 TL more** per month than workers in Region B.

Since the CI does not contain zero, we have evidence of a **statistically significant** wage gap between the two regions at the 5% level.

Caution: This does *not* mean the wage gap is *causal*. The regions may differ in education levels, industry composition, cost of living, etc. Establishing causality requires controlling for confounders—a task for regression analysis (Weeks 12–14).

Paired vs. Independent: quick decision rule

- Same subjects measured twice $\Rightarrow$ **Paired**
- Different subjects in each group $\Rightarrow$ **Independent**

Finite Population Correction (FPC)

Standard CIs assume sampling with replacement (or an infinite population). When sampling *without* replacement from a finite population of size N :

Finite Population Correction Factor

$$SE_{\text{corrected}} = \frac{S}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}}$$

The factor $\sqrt{\frac{N-n}{N-1}}$ is the FPC.

- When $n \ll N$ (e.g., $n/N < 0.05$), the FPC ≈ 1 and can be ignored
- When n is a large fraction of N , the FPC **shrinks** the SE—your sample contains a lot of the population, so there is less uncertainty!
- **Extreme case:** if $n = N$ (census), FPC = 0, SE = 0—no uncertainty at all
- **Rule of thumb:** Apply the FPC when $n/N > 0.05$ (sampling more than 5% of the population).

Summary: Common Mistakes to Avoid

The Errors Most Students Make

- ❶ **Confusing z and t :** Use z when σ is known (or n is very large); use t when σ is unknown and estimated by S
- ❷ **Wrong interpretation of CI:** “There is a 95% probability that μ is in this interval” is **WRONG**. Correct: “If we repeated sampling many times, 95% of the resulting intervals would contain μ ”
- ❸ **Forgetting degrees of freedom:** χ^2 uses $n - 1$; pooled t -test uses $n_1 + n_2 - 2$; Welch uses ν^*
- ❹ **Using CI for variance with non-normal data:** CI for σ^2 requires normality—CLT cannot be applied
- ❺ **Paired vs. independent confusion:** Same subjects $\Rightarrow$ paired; different subjects $\Rightarrow$ independent

Common Mistake: Interpreting Confidence Intervals

WRONG Interpretations

- “ μ has a 95% chance of being in (a, b) ”
- “95% of the data falls in (a, b) ”
- “The CI tells us the exact value of μ ”

CORRECT Interpretation

- “We are 95% **confident** that $\mu \in (a, b)$ ”
- “If we repeated the experiment many times, 95% of the constructed intervals would contain μ ”
- The randomness is in the **interval**, not in μ

Think of it this way: μ is a **fixed, unknown constant**. The CI is a **random interval** (because it depends on the sample). Before we collect data, there is a 95% probability the interval will “capture” μ . After we compute it, μ is either in it or not—we just don’t know which.

Review Problem 1: CLT and Sample Size

Problem: Monthly household electricity consumption in a city has mean $\mu = 250$ kWh and standard deviation $\sigma = 60$ kWh.

- (a) For a random sample of $n = 36$ households, find $\mathbb{P}(\bar{X} > 265)$.
- (b) How large should n be so that a 99% CI for μ has a margin of error of at most 10 kWh?

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Solution (a): By CLT, $\bar{X} \stackrel{\text{approx}}{\sim} N(250, 60^2/36) = N(250, 100)$.

$$Z = \frac{265 - 250}{60/\sqrt{36}} = \frac{15}{10} = 1.5$$

$$\mathbb{P}(\bar{X} > 265) = \mathbb{P}(Z > 1.5) = 1 - 0.9332 = \boxed{0.0668}$$

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Solution (b): $n \geq \left(\frac{z_{0.005} \cdot \sigma}{E}\right)^2 = \left(\frac{2.576 \times 60}{10}\right)^2 = (15.456)^2 = 238.9 \implies \boxed{n \geq 239}$

Review Problem 2: Confidence Interval for Variance

Problem: A pharmaceutical company tests the consistency of drug dosages. A sample of $n = 15$ tablets yields $S^2 = 4.2 \text{ mg}^2$. Assuming normality:

- (a) Construct a 90% CI for the population variance σ^2 .
- (b) The regulatory standard requires $\sigma^2 < 8 \text{ mg}^2$. Based on your CI, does the company meet the standard?

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Solution (a): $\nu = 14$. From $\chi^2(14)$ table:

$$\chi_{0.05, 14}^2 = 23.685, \quad \chi_{0.95, 14}^2 = 6.571$$

$$\text{CI} : \left(\frac{14 \times 4.2}{23.685}, \frac{14 \times 4.2}{6.571} \right) = \left(\frac{58.8}{23.685}, \frac{58.8}{6.571} \right) = \boxed{(2.483, 8.950)}$$

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(b) The upper bound (8.950) exceeds the regulatory limit of 8. Therefore, we **cannot conclusively say** the company meets the standard at the 90% confidence level. The interval includes values above 8, so more data (larger n) would be needed.

Review Problem 3: Two-Population Comparison

Problem: A labor economist compares hourly wages between male and female workers in a sector:

- Males: $n_1 = 40$, $\bar{X}_1 = 42.5$ TL, $S_1 = 8.2$ TL
- Females: $n_2 = 35$, $\bar{X}_2 = 38.0$ TL, $S_2 = 7.5$ TL

- Construct a 95% CI for $\mu_1 - \mu_2$.
- Is there evidence of a gender wage gap at the 5% significance level?

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(a) Construct a 95% CI for $\mu_1 - \mu_2$.

(b) Is there evidence of a gender wage gap at the 5% significance level?

Solution (a): $\bar{X}_1 - \bar{X}_2 = 4.5$, $SE = \sqrt{\frac{8.2^2}{40} + \frac{7.5^2}{35}} = \sqrt{1.681 + 1.607} = \sqrt{3.288} = 1.813$

$$\nu^* = \frac{(3.288)^2}{\frac{(1.681)^2}{39} + \frac{(1.607)^2}{34}} = \frac{10.81}{0.0724 + 0.0760} = \frac{10.81}{0.1484} \approx 72.8 \implies \nu^* = 72$$

$$t_{0.025, 72} \approx 1.993$$

$$95\% \text{ CI : } 4.5 \pm 1.993 \times 1.813 = 4.5 \pm 3.613 = \boxed{(0.887, 8.113)}$$

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$$\nu^* = \frac{(3.288)^2}{\frac{(1.681)^2}{39} + \frac{(1.607)^2}{34}} = \frac{10.81}{0.0724 + 0.0760} = \frac{10.81}{0.1484} \approx 72.8 \implies \nu^* = 72$$

$$t_{0.025, 72} \approx 1.993$$

$$95\% \text{ CI : } 4.5 \pm 1.993 \times 1.813 = 4.5 \pm 3.613 = \boxed{(0.887, 8.113)}$$

(b) Yes—the CI excludes zero, providing evidence of a statistically significant wage gap.

Review Problem 4: Proportion and Sample Size

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- (a) In a pilot study of $n = 200$ consumers, 68 expressed interest. Construct a 95% CI for the true proportion p of interested consumers.
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(b):

$$n = \left(\frac{1.96}{0.02} \right)^2 \times 0.34 \times 0.66 = 9604 \times 0.2244 = 2155.1 \implies \boxed{n \geq 2,156}$$

Further Reading

- Wackerly, Mendenhall & Scheaffer, Ch. 8: Estimation (Sections 8.5–8.9)
- Hogg, Tanis & Zimmerman, Ch. 6: Point Estimation & Ch. 7: Interval Estimation
- Devore & Berk, Ch. 7: Statistical Intervals Based on a Single Sample; Ch. 9: Inferences Based on Two Samples