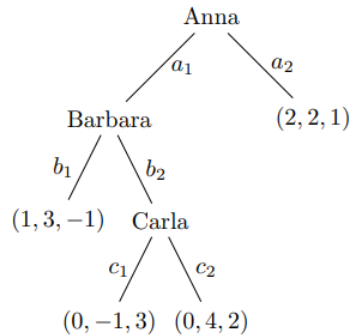


PROBLEM SET # 8

1. Anna, Barbara and Clara are playing the following extensive form game,



- (a) Write the game in its normal form.
 - (b) Find the pure strategy Nash equilibria of the game.
 - (c) Find the proper subgames of the game and write them in their normal forms.
 - (d) Find the Nash equilibria of such subgames.
 - (e) Find the subgame perfect Nash equilibria of the game.
2. A firm's foundation has to choose whether to contribute to University A or to University B. Each University is interested only in the contributions it receives. The foundation announces that the contribution will be decided by the following mechanism. University A is proposed a contribution of \$10,000 for itself and \$0 for University B. If University A accepts, the contribution is assigned. Otherwise the Foundation increases the total contribution to \$40,000 and asks University B whether it wants to keep all the \$40,000 for itself or whether it prefers that the contribution is equally shared between the two Universities (\$20,000 to A and \$20,000 to B).
- (a) Draw the extensive form of the game (i.e., the tree whose nodes represent when players play and whose branches represent their decisions).
 - (b) Find the subgame perfect equilibrium of the game.
 - (c) Represent the game in its normal form (i.e., the matrix whose rows represent University A's strategies and whose columns represent University B's strategies, and the cells containing players' payoffs for any possible combination of strategies).
 - (d) Find the pure-strategy Nash equilibria. Has one of the players a dominant or a dominated strategy?
3. Macrosoft has developed a new video game and it wants to launch it on the market. In particular the choice is between using an impacting advertising campaign or leaving the promotion to the communication between buyers and potential buyers ('word of mouth'). Macrosoft knows that the video game will last only 2 years on the market (as in 2 years newer and better video games will be introduced). It also knows that the sales do not depend on marketing strategy, but the temporal distribution of the sales do. The advertising campaign produces high sales in the first year but lower in the second due to market saturation. Using the word of mouth strategy, sales will be lower

in the first year and higher in the second year. The following table describes Microsoft's income depending on the strategy it adopts.

	Advertising	Word of mouth
Microsoft first year income	900,000	200,000
Microsoft second year income	100,000	800,000
Marketing cost	570,000	200,000

- (a) Draw the decision tree that represents Microsoft problem and derive Microsoft optimal decision.
 - (b) Draw the extensive form of the game(the 'game tree').
 - (c) Find the subgame perfect Nash equilibria of the game.
 - (d) What is Microsoft's strategy space?
 - (e) What is Microcorp's strategy space?
 - (f) What is the normal form of the game?
 - (g) Find the pure strategy Nash equilibria.

4. Companies A and B are considering developing a new commercial aircraft. Suppose that A is ahead in the development process and that B is considering whether to enter the competition. If B stays out, it earns zero profit and A, being the monopolist, earns a profit of \$1 billion. If B decides to enter and develop the rival airplane, then A has to decide whether to accommodate B peaceably, or to wage a price war. In the event of peaceful competition, each firm will make a profit of \$300 million. If there is a price war, each will lose \$100 million
 - (a) Draw the game tree ('extensive form') for this game
 - (b) What is the subgame perfect Nash equilibrium?
 - (c) How many strategies does each player have?
 - (d) Write the normal form of the game.
 - (e) Find all Nash equilibria of this game.

5. Consider the game in which the following are commonly known. First, Ann chooses between actions a and b. Then, with probability 1/3, Bob observes which action Ann has chosen and with probability 2/3 he does not observe the action she has chosen. In all cases (regardless of whether he has observed Ann chose a, or he has observed Ann chose b, or he has not observed any action), Bob chooses between actions α and β . The payoff of each player is 1 after (a, α) and (b, β) and 0 otherwise.
 - (a) Write the above game in extensive form.
 - (b) Write the above game in normal form.

6. Consider the following variation of the above game. First, Ann chooses between actions a and b. Then, Bob decides whether to observe the chosen action of Ann or not, by choosing between the actions Open and Shut, respectively. In all cases, Bob then chooses between actions α and β . The payoff of Ann is 1 after (a, α) and (b, β)

and 0 otherwise, regardless of whether Bob chooses Open or Shut. The payoff of Bob is equal to the payoff of Ann if he has chosen Shut, and his payoff is equal to the payoff of Ann minus $1/2$ if he has chosen Open.

- (a) Write the above game in extensive form
- (b) Write the above game in normal form.