

2025-2026

ECON 205

Problem Set 4- Answers

- 1) If Y is a continuous random variable with mean μ and variance σ^2 , a and b are constants, prove the following:

$$a) \quad E(aY+b) = aE(Y)+b = a\mu+b$$

$$E(aY + b) = \int_{-\infty}^{\infty} (ay + b)f(y)dy \quad (1)$$

$$E(aY + b) = a \int_{-\infty}^{\infty} y f(y)dy + b \int_{-\infty}^{\infty} f(y)dy \quad (2)$$

$$E(aY + b) = a E(Y) + b,$$

$$\text{where } \int_{-\infty}^{\infty} y f(y)dy = E(Y) \text{ and } \int_{-\infty}^{\infty} f(y)dy = 1 \quad (3)$$

$$E(aY + b) = a\mu + b \quad (4)$$

$$b) \quad V(aY+b) = a^2V(Y) = a^2\sigma^2$$

$$V(aY + b) = E[(aY + b)^2] - [E(aY + b)]^2 \quad (5)$$

$$V(aY + b) = E[(a^2Y^2 + 2abY + b^2)] - [(aE(Y) + b)]^2 \quad (6)$$

$$V(aY + b) = a^2E(Y^2) + 2abE(Y) + b^2 - (a^2E(Y)^2 + 2abE(Y) + b^2) \quad (7)$$

$$V(aY + b) = a^2E(Y^2) - a^2E(Y)^2 \quad (8)$$

Rearrange the equation (8), then

$$V(aY + b) = a^2[E(Y^2) - E(Y)^2] \quad (9)$$

$$V(aY + b) = a^2V(Y) = a^2\sigma^2 \quad (10)$$

2) Mrs. Brown is pregnant and due in 100 days. Suppose that the probability density distribution function for having a child is approximately normal with mean 100 and standard deviation 8. Mrs. Brown has a business trip and will return in 85 days and have to go on another business trip in 107 days.

- a. What is the probability that the birth will occur before her second trip?
- b. What is the probability that the birth will occur after she returns from her first business trip?
- c. What is the probability that she will be here for the birth?

SOLUTION

a) $P(X < 107)$ is asked.

Compute the z-score

$$z = \frac{107 - 100}{8} = 0.88$$

$P(Z < 0.88) = 0.8106$ from the z-table

Hence, there is an 81% probability that the baby will be born before the second trip

b) Now, we want $P(X > 85)$

z-score:

$$z = \frac{85 - 100}{8} = -1.88$$

$P(Z < -1.88) = 0.0301$

$P(Z > -1.88) = 1 - 0.0301 = 0.9699$

Hence, there is a 97% probability that the baby will be born after the first trip

c) $P(85 < X < 107)$

$P(85 < X < 107) = P(X < 107) - P(X < 85) = 81\% - 3\% = 78\%$

Hence, there is 78% probability that Mrs Brown will make it to the birth

3) Scores on an examination are assumed to be normally distributed with mean 78 and variance 36.

a) What is the probability that a person taking the examination scores higher than 72?

$$\mu = 78 \text{ and } \sigma^2 = 36, \sigma = 6$$

$$P(Y > 72) = P\left(Z > \frac{72 - 78}{6}\right) \quad (1)$$

$$P(Z > -1) = 0.8413 \quad (2)$$

- b) Suppose that students scoring in the top 10% of this distribution are to receive an A grade. What is the minimum score a student must achieve to earn an A grade?

We seek c such that $P(Y > c) = 0.1$. For standard normal, $P(Z > z_0) = 0.1$ when $z_0 = 1.28$. Therefore, $c = 78 + (1.28)6 = 85.68$.

- c) What must be the cut-off point for passing the examination if the examiner wants only the top 28.1% of all scores to be passing?

We seek c such that $P(Y > c) = 0.281$. For standard normal, $P(Z > z_0) = 0.281$ when $z_0 = 0.58$. Therefore, $c = 78 + (0.58)6 = 81.48$.

- d) Approximately what proportion of students have scores 5 or more points above the score that cuts off the lowest 25%?

For the standard normal, $P(Z < -0.67) = 0.25$. Therefore, the score that cuts off the lowest 25% is given by $(-0.67)6 + 78 = 73.98$.

We need to find the proportion of students who have scores 78.98 or more.

$$P(X \geq 78.98) = P\left(Z \geq \frac{78.98 - 78}{6}\right) = P(Z \geq 0.163) \quad (3)$$

$$P(Z \geq 0.163) = 1 - P(Z < 0.163) \quad (4)$$

$$P(X \geq 78.98) = 1 - 0.5636 = 0.4364 \quad (5)$$

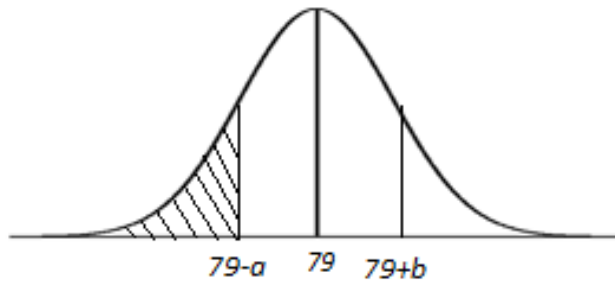
- e) If it is known that a student's score exceeds 72, what is the probability that his or her score exceeds 84?

$$P(Y > 84 | Y > 72) = \frac{P(Y > 84)}{P(Y > 72)} \quad (6)$$

$$P(Y > 84 | Y > 72) = \frac{P(Z > 1)}{P(Z > -1)} \quad (7)$$

$$P(Y > 84 | Y > 72) = \frac{0.1587}{0.8413} = .1886 \quad (8)$$

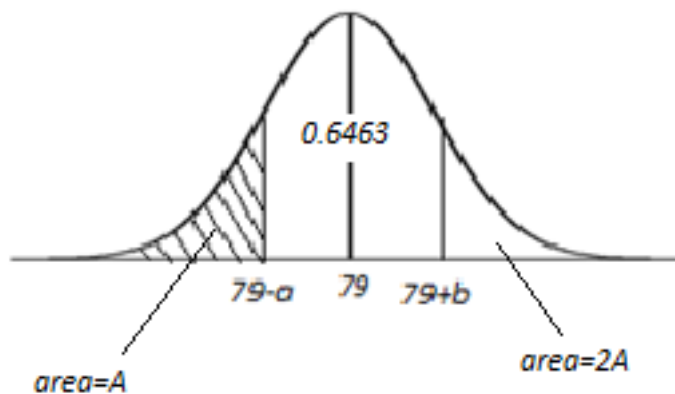
- 4) Let X be a normally distributed random variable with mean 79. Suppose it is known that $P(79-a \leq X \leq 79+b) = 0.6463$.



It is given that $P(X \geq 79+b) = 2P(X \leq 79-a)$

- Calculate the shaded area.
- Find the value of b .

SOLUTION



$$3A + 0.6463 = 1$$

- $A = 0.1179$
- $2A = 0.2358$
From the z-table,
 $z = 0.72$ Since $z = \frac{(79+b)-79}{\sigma}$, we obtain $b = 0.72\sigma$.

- 5) The weekly repair cost, X , for a certain machine has a probability density function given by:

$$f(x) = \begin{cases} cx(1-x), & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

with measurements in \$100.

- Find the value of c that makes this function a valid density function.
- Find the cumulative distribution function of X .
- What is the probability that repair costs will exceed \$75 during a week?
- What is the probability that the repair costs will exceed \$75 during a week given that they will exceed \$50?

SOLUTION

- a) Find the value of c that makes this function a valid density function.

For any pdf, $\int_{-\infty}^{\infty} f(x)dx = 1$

$$\int_0^1 cx(1-x)dx = c \int_0^1 (x - x^2)dx = c \left(\frac{x^2}{2} - \frac{x^3}{3} \right) \Bigg|_{x=0}^{x=1} = 1$$

$$c \left(\frac{1^2}{2} - \frac{1^3}{3} \right) - c \left(\frac{0^2}{2} - \frac{0^3}{3} \right) = \frac{c}{6} = 1$$

$$c=6$$

- b) Find the cumulative distribution function of X .

For $0 \leq x \leq 1$

$$F(x) = \int_0^x 6z(1-z)dz = 3x^2 - 2x^3$$

$$F(x) = \begin{cases} 0, & x < 0 \\ 3x^2 - 2x^3, & 0 \leq x < 1 \\ 1, & x \geq 1 \end{cases}$$

- c) What is the probability that repair costs will exceed \$75 during a week?

$$\text{Find } P(X > 0.75) = 1 - P(X \leq 0.75) = 1 - F(0.75) = 1 - [3(0.75)^2 - 2(0.75)^3] \cong 0.16$$

- d) What is the probability that the repair costs will exceed \$75 during a week given that they will exceed \$50?

$$P(X > 0.75 | X > 0.5) = \frac{P(X > 0.75 \cap X > 0.5)}{P(X > 0.5)} = \frac{P(X > 0.75)}{P(X > 0.5)} = \frac{1 - [3(0.75)^2 - 2(0.75)^3]}{1 - [3(0.5)^2 - 2(0.5)^3]} \cong 0.3$$

6) Suppose X is a continuous random variable with density function:

$$f(x) = \begin{cases} \frac{1}{2} x^2 e^{-x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Find the moment generation function.

The moment generation function can be written as:

$$M_x(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{xt} f(x) \cdot dx$$

$$M_x(t) = E(e^{tX}) = \int_0^{\infty} e^{xt} * \frac{1}{2} * x^2 * e^{-x} * dx = \frac{1}{2} \int_0^{\infty} x^2 e^{(t-1)x} dx \quad (1)$$

Integrate by parts: $\int f g' = f g - \int f' g$, where $f = x^2$, $g' = e^{(t-1)x}$ and

$$f' = 2x, \quad g = \frac{e^{(t-1)x}}{t-1}$$

$$M_x(t) = \frac{1}{2} \left[\frac{x^2 e^{(t-1)x}}{(t-1)} \Big|_0^{\infty} - \int_0^{\infty} \frac{2x e^{(t-1)x}}{t-1} dx \right] \quad (2)$$

$$M_x(t) = \frac{1}{2} \left[\frac{x^2 e^{(t-1)x}}{(t-1)} \Big|_0^{\infty} - \frac{2}{t-1} \int_0^{\infty} x e^{(t-1)x} dx \right] \quad (3)$$

Now, integrate by parts: $\int f g' = f g - \int f' g$, where $f = x$, $g' = e^{(t-1)x}$ and

$$f' = 1, \quad g = \frac{e^{(t-1)x}}{t-1}$$

$$\int_0^{\infty} x e^{(t-1)x} dx = \frac{x e^{(t-1)x}}{t-1} \Big|_0^{\infty} - \int_0^{\infty} \frac{e^{(t-1)x}}{t-1} dx \quad (4)$$

$$\int_0^{\infty} x e^{(t-1)x} dx = \frac{x e^{(t-1)x}}{t-1} \Big|_0^{\infty} - \frac{e^{(t-1)x}}{(t-1)^2} \Big|_0^{\infty} \quad (5)$$

Plug (5) in (3)

$$M_x(t) = \frac{1}{2} \left[\frac{x^2 e^{(t-1)x}}{(t-1)} \Big|_0^{\infty} - \frac{2}{t-1} \left[\frac{x e^{(t-1)x}}{t-1} \Big|_0^{\infty} - \frac{e^{(t-1)x}}{(t-1)^2} \Big|_0^{\infty} \right] \right] \quad (6)$$

$$M_x(t) = \frac{1}{2} \left[\left(\frac{x^2}{t-1} - \frac{2x}{(t-1)^2} + \frac{2}{(t-1)^3} \right) e^{(t-1)x} \right]_0^{\infty} = \frac{1}{(1-t)^3} \quad (7)$$

Where the last equality holds for $t < 1$. So the mgf is defined for $t < 1$.

- 7) A 12-in. bar clamped at both ends is subjected to an increasing amount of stress until it snaps. Let Y = the distance from the left end at which the break occurs. Suppose Y has pdf

$$f(y) = \frac{y}{24} \left(1 - \frac{y}{12}\right) \quad 0 \leq y \leq 12$$

Compute the following:

- The cumulative distribution function (CDF) of Y .
- $P(Y \leq 4)$, $P(Y > 6)$ and $P(4 \leq Y \leq 6)$.
- $E(Y)$, $E(Y^2)$, and $V(Y)$.
- The probability that the break point occurs more than 2 inches from the expected break point.
- The expected length of the shorter segment when the break occurs.

SOLUTION

- a. For

$$0 \leq y \leq 12$$

$$F(y) = \frac{1}{24} \int_0^y \left(u - \frac{u^2}{12}\right) du = \frac{1}{24} \left(\frac{u^2}{2} - \frac{u^3}{36}\right) \Big|_0^y = \frac{y^2}{48} - \frac{y^3}{864}. \text{ Thus}$$

$$F(y) = \begin{cases} 0 & y < 0 \\ \frac{y^2}{48} - \frac{y^3}{864} & 0 \leq y \leq 12 \\ 1 & y > 12 \end{cases}$$

- b. $P(Y \leq 4) = F(4) = .259$. $P(Y > 6) = 1 - F(6) = .5$.
 $P(4 \leq Y \leq 6) = F(6) - F(4) = .5 - .259 = .241$.

c. $E(Y) = \int_0^{12} y \cdot \frac{1}{24} y \left(1 - \frac{y}{12}\right) dy = \frac{1}{24} \int_0^{12} \left(y^2 - \frac{y^3}{12}\right) dy = \frac{1}{24} \left[\frac{y^3}{3} - \frac{y^4}{48}\right]_0^{12} = 6 \text{ inches.}$
 $E(Y^2) = \frac{1}{24} \int_0^{12} \left(y^3 - \frac{y^4}{12}\right) dy = 43.2$, so $V(Y) = 43.2 - 36 = 7.2$.

- d. $P(Y < 4 \text{ or } Y > 8) = 1 - P(4 \leq Y \leq 8) = 1 - [F(8) - F(4)] = .518$.

- e. The shorter segment has length equal to $\min(Y, 12 - Y)$, and

$$\begin{aligned} E[\min(Y, 12 - Y)] &= \int_0^{12} \min(y, 12 - y) \cdot f(y) dy = \int_0^6 \min(y, 12 - y) \cdot f(y) dy \\ &+ \int_6^{12} \min(y, 12 - y) \cdot f(y) dy = \int_0^6 y \cdot f(y) dy + \int_6^{12} (12 - y) \cdot f(y) dy = \frac{90}{24} = 3.75 \text{ inches.} \end{aligned}$$

- 8) The operator of a pumping station has observed that demand for water during early afternoon hours has an approximately exponential distribution with mean 100 cfs (cubic feet per second).
- Find the probability that the demand will exceed 200 cfs during the early afternoon on a randomly selected day.
 - What water-pumping capacity should the station maintain during early afternoons so that the probability that demand will exceed capacity on a randomly selected day is only .01?

SOLUTION

Let Y = water demand in the early afternoon. Then, Y is exponential with $\beta = 100$ cfs.

$$\text{a. } P(Y > 200) = \int_{200}^{\infty} \frac{1}{100} e^{-y/100} dy = e^{-2} = .1353.$$

b. We require the 99th percentile of the distribution of Y :

$$P(Y > \phi_{.99}) = \int_{\phi_{.99}}^{\infty} \frac{1}{100} e^{-y/100} dy = e^{-\phi_{.99}/100} = .01. \text{ So, } \phi_{.99} = -100 \ln(.01) = 460.52 \text{ cfs.}$$

- 9) The time till failure of an electronic component has an exponential distribution and it is known that 10% of components have failed by 1000 hours. What is the probability that a component is still working after 5000 hours?

SOLUTION

Let Y be a random variable represents the time until failure occurs in hours

$$f(y) = \begin{cases} ve^{-vy}, & 0 \leq y, \\ 0, & \text{elsewhere} \end{cases}$$

First, we need to find v . To find it, we can use the knowledge in the question (e.g. it is known that...).

$$P(Y \leq 1000) = \int_0^{1000} ve^{-vy} dy = -e^{-vy} \Big|_{x=0}^{x=1000} = 1 - e^{-1000v}$$

$$P(Y \leq 1000) = 0.1 = 1 - e^{-1000v} \rightarrow v = 1.05 * 10^{-4}$$

We want to find $P(Y > 5000)$

$$P(Y > 5000) = \int_{5000}^{\infty} ve^{-vy} dy = -e^{-vy} \Big|_{y=5000}^{y=\infty} = e^{-5000v} \approx 0.59$$

10) Suppose that when a type of transistor is subjected to an accelerated life test, the lifetime X (in weeks) has a gamma distribution with mean 24 weeks and standard deviation 12 weeks.

- What is the probability that a transistor will last between 12 and 24 weeks?
- What is the probability that a transistor will last at most 24 weeks? Is the median of the lifetime distribution less than 24? Why or why not?
- What is the 99th percentile of the lifetime distribution?

SOLUTION

Let X be the lifetime of transistor. And, we know X has a gamma distribution, and the expected value and standard deviation.

$$\mu = \alpha * \beta \text{ \& } \sigma^2 = \alpha * \beta^2$$

$$\alpha\beta = 24 \text{ \& } \alpha\beta^2 = 144$$

Now we have two unknowns and two equations, we can solve it.

$$\frac{\alpha\beta}{\alpha\beta^2} = \frac{24}{144} \rightarrow \beta = 6 \text{ \& } \alpha = 4$$

a)

Now, we can calculate the probability by using the Table A.4 in the textbook which gives us cdf of X .

$$P(12 < X < 24) = F\left(\frac{24}{6}; 4\right) - F\left(\frac{12}{6}; 4\right) = F(4; 4) - F(2; 4)$$

Table A.4 The Incomplete Gamma Function

$x \backslash \alpha$	1	2	3	4	5
1	.632	.264	.080	.019	.004
2	.865	.594	.323	.143	.053
3	.950	.801	.577	.353	.185
4	.982	.908	.762	.567	.371
5	.993	.960	.875	.735	.560

$$P(12 < X < 24) = 0.567 - 0.143 = 0.424$$

b)

$$P(X \leq 24) = F\left(\frac{24}{6}; 4\right) = F(4; 4) = 0.567$$

Table A.4 The Incomplete Gamma Function

$x \backslash \alpha$	1	2	3	4	5
1	.632	.264	.080	.019	.004
2	.865	.594	.323	.143	.053
3	.950	.801	.577	.353	.185
4	.982	.908	.762	.567	.371
5	.993	.960	.875	.735	.560

The median of the lifetime distribution is less than 24, since $P(X \leq 24)$ is equal to 0.567. A number that smaller than 24 can give the probability of 0.5, and this number can be the median of lifetime distribution.

c)

We are trying to find $P(X \leq c) = 0.99$

$$P(X \leq c) = F\left(\frac{c}{6}; 4\right) = 0.99$$

$x \backslash \alpha$	1	2	3	4
1	.632	.264	.080	.019
2	.865	.594	.323	.143
3	.950	.801	.577	.353
4	.982	.908	.762	.567
5	.993	.960	.875	.735
6	.998	.983	.938	.849
7	.999	.993	.970	.918
8	1.000	.997	.986	.958
9		.999	.994	.979
10		1.000	.997	.990
11			.999	.995

$$F(10; 4) = 0.99 \rightarrow \frac{c}{6} = 10. \text{ So, } c = 6 * 10 = 60$$

- 11) The weekly amount of downtime Y (in hours) for an industrial machine has approximately a gamma distribution with $\alpha = 3$ and $\beta = 2$. The loss L (in dollars) to the industrial operation as a result of this downtime is given by $L = 30Y + 2Y^2$. Find the expected value and variance of L .

SOLUTION

Let Y have a gamma distribution with $\alpha = 3$ and $\beta = 2$. Then, the loss $L = 30Y + 2Y^2$.

Then,

$$E(L) = E(30Y + 2Y^2) = 30E(Y) + 2E(Y^2) = 30(6) + 2(12 + 6^2) = 276,$$

$$V(L) = E(L^2) - [E(L)]^2 = E(900Y^2 + 120Y^3 + 4Y^4) - 276^2.$$

$$E(Y^3) = \int_0^{\infty} \frac{y^5}{16} e^{-y/2} = 480$$

$$E(Y^4) = \int_0^{\infty} \frac{y^6}{16} e^{-y/2} = 5760$$

$$\text{Thus, } V(L) = 900(48) + 120(480) + 4(5760) - 276^2 = 47,664.$$

- 12) The lifetime of X (in hundreds of hours) of a light bulb has Weibull distribution with parameters $\alpha=2$ and $\beta=3$.

- a) Find $E(X)$ and $V(X)$
- b) Compute $P(X \leq 6)$.
- c) Compute $P(2.8 \leq X \leq 4)$.
- d) Compute $P(X \geq 3)$.

SOLUTION

Let X denote the lifetime (in hundreds of hours) of light bulb.

Using the pdf definition of the Weibull Distribution with parameters $\alpha > 0$ and $\beta > 0$,

$$f(x; \alpha, \beta) = \begin{cases} \frac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{-(x/\beta)^\alpha}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

The cumulative distribution function of X is

$$F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-(x/\beta)^\alpha}, & x \geq 0 \end{cases}$$

a)

The expected value of X

$$E(X) = \beta \Gamma\left(1 + \frac{1}{\alpha}\right) \quad (3)$$

$$E(X) = 3\Gamma\left(1 + \frac{1}{2}\right) = 3\Gamma\left(\frac{3}{2}\right) \quad (4)$$

For this part of the question, we use two important properties of Gamma function.

$$\diamond \text{ For any } \alpha > 1, \Gamma(\alpha) = (\alpha - 1)\Gamma(\alpha - 1)$$

$$\diamond \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$E(X) = 3 * \left(\frac{3}{2} - 1\right) \Gamma\left(\frac{3}{2} - 1\right) = 3 * \frac{1}{2} \Gamma\left(\frac{1}{2}\right) \quad (5)$$

$$E(X) = \frac{3}{2} * \sqrt{\pi} = \frac{3}{2} * 1.7725 = 2.6587 \quad (6)$$

The variance of X ,

$$V(X) = \sigma^2 = \beta^2 \left\{ \Gamma\left(1 + \frac{2}{\alpha}\right) - \left[\Gamma\left(1 + \frac{1}{\alpha}\right) \right]^2 \right\}$$

$$V(X) = 3^2 \left\{ \Gamma\left(1 + \frac{2}{2}\right) - \left[\Gamma\left(1 + \frac{1}{2}\right) \right]^2 \right\}$$

Recall another property of the Gamma function,

$$\diamond \text{ For any positive integer, } n, \Gamma(n) = (n - 1)!$$

$$V(X) = 9\{\Gamma(2) - [\Gamma(3/2)]^2\} = 9\left[1 - \left(\frac{1}{2} * \Gamma\left(\frac{1}{2}\right)\right)^2\right]$$

$$V(X) = 9\left[1 - \left(\frac{\sqrt{\pi}}{2}\right)^2\right] = 9\left[1 - \left(\frac{\sqrt{3.1416}}{2}\right)^2\right] = 1.931246$$

b)

$$P(X \leq 6) = F(6)$$

$$P(X \leq 6) = 1 - e^{-\left(\frac{6}{3}\right)^2} = 1 - e^{-(2)^2} = 1 - e^{-4}$$

$$P(X \leq 6) = 1 - 0.0183 = 0.9817$$

c)

$$P(2.8 \leq X \leq 4) = F(4) - F(2.8)$$

$$P(2.8 \leq X \leq 4) = \left[1 - e^{-\left(\frac{4}{3}\right)^2}\right] - \left[1 - e^{-\left(\frac{2.8}{3}\right)^2}\right] = -e^{-1.78} + e^{-0.87}$$

$$P(2.8 \leq X \leq 4) = -0.1686 + 0.4190 = 0.2504$$

d)

$$P(X \geq 3) = 1 - P(X < 3) = 1 - F(3)$$

$$P(X \geq 3) = 1 - \left[1 - e^{-\left(\frac{3}{3}\right)^2} \right] = e^{-(1)^2} = 0.3679$$

Table A.4 The incomplete gamma function

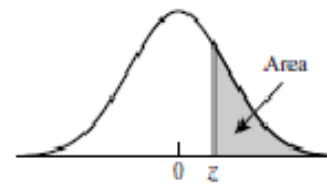
$G(x; \alpha) = \int_0^x \frac{1}{\Gamma(\alpha)} y^{\alpha-1} e^{-y} dy$										
x	α									
	1	2	3	4	5	6	7	8	9	10
1	.632	.264	.080	.019	.004	.001	.000	.000	.000	.000
2	.865	.594	.323	.143	.053	.017	.005	.001	.000	.000
3	.950	.801	.577	.353	.185	.084	.034	.012	.004	.001
4	.982	.908	.762	.567	.371	.215	.111	.051	.021	.008
5	.993	.960	.875	.735	.560	.384	.238	.133	.068	.032
6	.998	.983	.938	.849	.715	.554	.394	.256	.153	.084
7	.999	.993	.970	.918	.827	.699	.550	.401	.271	.170
8	1.000	.997	.986	.958	.900	.809	.687	.547	.407	.283
9		.999	.994	.979	.945	.884	.793	.676	.544	.413
10		1.000	.997	.990	.971	.933	.870	.780	.667	.542
11			.999	.995	.985	.962	.921	.857	.768	.659
12			1.000	.998	.992	.980	.954	.911	.845	.758
13				.999	.996	.989	.974	.946	.900	.834
14				1.000	.998	.994	.986	.968	.938	.891
15					.999	.997	.992	.982	.963	.930

Table A.3 Standard Normal Curve Areas (cont.)

$\Phi(z) = P(Z \leq z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Normal Curve Areas
Standard normal probability in right-hand
tail (for negative values of z , areas are found by symmetry)



Second decimal place of z										
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014