

2025-2026
ECON 205

Problem Set 2

- 1) Suppose that $P(A_1 \cap A_2 \cap A_3) > 0$ and show that

$$P(A_1 \cap A_2 \cap A_3 \cap A_4) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2)P(A_4|A_1 \cap A_2 \cap A_3)$$

- 2) A, B, and C are mutually independent events such that

$$P(A) = 0.4, P(B) = 0.25, P(C) = 0.35$$

$$P[(A' \cap B') \cup C] = ?$$

- 3) Many public schools are implementing a “no-pass, no-play” rule for athletes. Under this system, a student who fails a course is disqualified from participating in extracurricular activities during the next grading period. Suppose that the probability is .15 that an athlete who has not previously been disqualified will be disqualified next term. For athletes who have been previously disqualified, the probability of disqualification next term is .5. If 30% of the athletes have been disqualified in previous terms, what is the probability that a randomly selected athlete will be disqualified during the next grading period?
- 4) Consider independent trials consisting of rolling a pair of fair dice, over and over. What is the probability that a sum of 6 appears before a sum of 9?
- 5) Let Y be a discrete random variable with mean μ and variance σ^2 . If a and b are constants, prove that $V(aY+b)=a^2V(Y)=a^2\sigma^2$.

6) Let Y be a random variable with $p(y)$ and $p(0)=0.2$, $p(1)=0.3$, $p(2)=0.4$, $p(4)=0.1$. Find $E(Y)$, $E(Y^2)$ and $V(Y)$.

7) A contractor is interested in the total cost of a Project on which she intends to bid. She estimates that materials will cost \$25,000 and that her labor will be \$900 per day. If the project takes X days to complete, the total labor cost will be $900X$, and the total cost of project (in dollars) will be as follows:

$$C = 25,000 + 900X$$

Using her experience, the contractor forms probabilities of likely completion times for the project.

- Find the mean and variance for completion time X .
- Find $E(1/X)$ and $E(\frac{1}{X} - 1)$.
- Find the mean and variance for total cost C .

Probability Distribution for Completion Times

COMPLETION TIMES X (DAYS)	10	11	12	13	14
Probability	0.1	0.3	0.3	0.2	0.1

8) Two gamblers bet \$1 each on the successive tosses of a coin. Each has a bank of \$6. What is the probability that they

- break even after six tosses of the coin?
- one player -say, Jones- wins all the money on the tenth toss of the coin?

- 9) Airlines sometimes overbook flights. Suppose that for a plane with 50 seats, 55 passengers have tickets. Define the random variables Y as the number of ticketed passengers who show up for the flight. The probability mass function of Y appears in the accompanying table.

y	45	46	47	48	49	50	51	52	53	54	55
$p(y)$	.05	.10	.12	.14	.25	.17	.06	.05	.03	.02	.01

- What is the probability that the flight will accommodate all ticketed passengers who show up?
- What is the probability that not all ticketed passengers who show up can be accommodated?
- If you are the first person on the standby list consisting of people without tickets (which means you will be the first one to get on the plane if there are any seats available after all ticketed passengers have been accommodated), what is the probability that you will be able to take the flight? What is this probability if you are the third person on the standby list?

- 10) Suppose that Y possesses the probability mass function:

$$f(y) = \begin{cases} cy, & y = 2, 4, 6, 8, 10 \\ 0, & \text{otherwise} \end{cases}$$

- Find the value of c that makes $f(y)$ a probability function.
 - Find $P(Y \geq 3)$
- 11) Let $m(t) = \left(\frac{1}{6}\right)e^t + \left(\frac{2}{6}\right)e^{2t} + \left(\frac{3}{6}\right)e^{3t}$ be the moment-generating function of a random variable Y . Find the following:
- $E(Y)$
 - $V(Y)$