

① call the three choices: q^A, q^B, q^C .

(a) Both q^A and q^B are affordable in A and therefore q^A is revealed preferred to q^B . Both q^B and q^C are affordable in B and therefore q^B is revealed preferred to q^C . Both q^A and q^C are affordable in C and therefore q^C is revealed preferred to q^A . There is no violation of WARP but the revealed preference relation is not transitive, violating SAEP.

Regime	Price			Consumption		
	P_1	P_2	P_3	q_1	q_2	q_3
A	1	1	2	1	0	0
B	2	1	1	0	1	0
C	1	2	1	0	0	1

P/q	q^A	q^B	q^C
P_1	1	1	2
P_2	2	1	1
P_3	1	2	1

— $q^A > q^B$

— $q^B > q^C$

— $q^C > q^A$

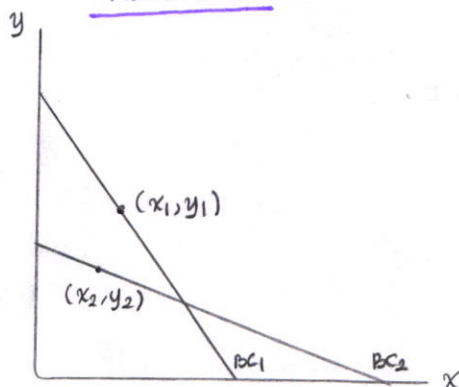
bundle

choices of the consumer under each price regime

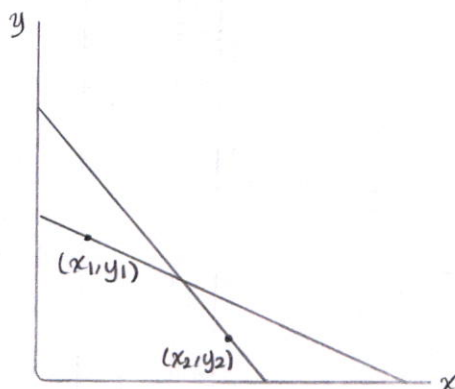
These preferences would violate the WARP, if, for example, A would be affordable under P_2 , then $q^B > q^A$ & since w/ prices P_1 $q^A > q^B$.

(b) No, since SAEP is equivalent to compatibility with optimizing behavior.

Reminder About WARP:



• The consumer was consuming (x_1, y_1) when (x_2, y_2) was affordable. But when s/he chooses (x_2, y_2) since (x_1, y_1) was not affordable.



• (x_1, y_1) is chosen when (x_2, y_2) is affordable
• (x_2, y_2) is chosen when (x_1, y_1) is affordable.

} violates WARP

— choices are not consistent
 $(x_1, y_1) > (x_2, y_2)$
 $(x_2, y_2) > (x_1, y_1)$

• $(x_1, y_1) \neq (x_2, y_2)$

• If (x_1, y_1) is directly revealed preferred to (x_2, y_2) & $(x_1, y_1) \neq (x_2, y_2)$ then it can't happen that (x_2, y_2) is directly RP (x_1, y_1)

$$P_1 x_1 + P_2 y_1 \geq P_1 x_2 + P_2 y_2$$

It can't be the case that

$$q_1 x_2 + q_2 y_2 \geq q_1 x_1 + q_2 y_1$$

②

(a) Denote the consumption of good 2 in year 2 by x_2^2 .

If the consumer's consumption bundle in year 1 is revealed preferred to that in year 2 then the year 2 bundle must be affordable with year 1 prices and income. so,

$$(100)(100) + (100)(100) \geq (100)(120) + (100)x_2^2$$

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Year 1 income $x_2^2 \leq 80$ └ year 2 bundle with year 1 prices

(b) If the consumer's consumption bundle in year 2 is revealed preferred to that in year 1 then the year 1 bundle must be affordable with year 2 prices and income.

so;

$$(100)(120) + 80x_2^2 \geq (100)(100) + (80)(100) \quad \text{OR}$$

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Year 2 income $x_2^2 \geq 75$. └ year 1 bundle with year 2 prices

(c) This requires that the inequalities in (a) and (b) both hold, so $75 \leq x_2^2 \leq 80$.

$$\text{└ } (x_1^1, x_2^1) \succ (x_1^2, x_2^2) \wedge (x_1^2, x_2^2) \succ (x_1^1, x_2^1)$$

(d) $x_2^2 \leq 75$ — b/w $75 \leq x_2^2 \leq 80$
WARP is violated.
In year 2, the amount of good 1 ↑, so income should decrease.

Inferior
Good $\Rightarrow \frac{\partial x_1}{\partial m} < 0$
└ we know that the amount of x_1 ↑ in year 2. So, the consumer must have a lower income than year 1.

(e) $80 < x_2^2 \leq 100$

└ As $I \uparrow$ $x_2^2 \downarrow$

$$(100)(100) + (100)(100) < (100)(120) + (100)x_2^2$$

$$x_2^2 > 80$$

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It can be at most 100 since $x_2^1 = 100$ units.

$$(100)(100) + (100)(100) > (100)x_2^2 + (100)(120)$$

at this point WARP is violated b/c $80 < x_2^2 \leq 100$

③ (a)

i. $(P_1, P_2) = (6, 3)$ and $A = (9, 18)$

$$I_1 = 6 \cdot 9 + 3 \cdot 18 = 108$$

$$6 \cdot 8 + 3 \cdot 14 = 90 \text{ (Bundle B with A prices)}$$

ii. $(P_1, P_2) = (1, 2)$ and $B = (8, 14)$

$$I_2 = 1 \cdot 8 + 2 \cdot 14 = 36$$

$$1 \cdot 9 + 2 \cdot 18 = 45 \text{ (Bundle A with B prices)}$$

Both A and B bundles are affordable with prices and income in (i) and A is revealed preferred to B. In (ii), only B is affordable and A is not affordable. Thus, there is no violation of WARP, since when B is chosen A is not affordable.

(b) $(P_1, P_2) = (6, 2)$ and $A = (6, 6) \Rightarrow I_1 = 6 \cdot 6 + 2 \cdot 6 = 48$ $\neg$ B is not affordable at these prices.
 $6 \cdot 10 + 2 \cdot 0 = 60$ (Bundle B with A prices)

$(P_1, P_2) = (2, 5)$ and $B = (10, 0) \Rightarrow I_2 = 2 \cdot 10 + 5 \cdot 0 = 20$ $\neg$ A is not affordable at these prices.
 $2 \cdot 6 + 5 \cdot 6 = 42$ (Bundle A w/ B prices)

Neither of the bundles are revealed preferred to the other. No evidence that she violate WARP.

(c)

	A	B	C
Bundle:	(7, 8)	(9, 4)	(7, 5)
Prices:	(16, 4)	(5, 6)	(3, 3)

Bundles \ Prices	A (7, 8)	B (9, 4)	C (7, 5)
(16, 4)	144	160	132
(5, 6)	83	69	65
(3, 3)	45	39	36

- Bundle B is not affordable in Country A whereas C is affordable $\Rightarrow$ A is revealed preferred to C.
- Bundle A is not affordable in Country B, whereas C is affordable $\Rightarrow$ B is revealed preferred to C.
- Bundle A and B are not affordable in country C. There is no violation of WARP.

(d)

Bundles \ Prices	A (8, 20)	B (10, 13)	C (14, 11)
(4, 2)	72	66	78
(2, 4)	96	72	72
(P_1'', P_2'')	$8P_1'' + 20P_2''$	$10P_1'' + 13P_2''$	$14P_1'' + 11P_2''$

- A is revealed preferred to B.
 - B is revealed preferred to C.
 (C is affordable with prices P_1', P_2')
- By transitivity $\Rightarrow$ A is indirectly revealed preferred to C.

WARP requires $\Rightarrow$ If some bundle x is directly revealed preferred to bundle y , it cannot happen that y is directly revealed preferred to x .

SARP requires $\Rightarrow$ If bundle x is (either directly or indirectly) revealed preferred to bundle y , then it cannot happen that y is directly or indirectly revealed preferred to x .

• We know that B is revealed preferred to C . So, according to both WARP and SARP, we cannot have C is revealed preferred to B which implies that when C is chosen, B should be out of our budget set (not affordable).

Thus, $10P_1'' + 13P_2'' > 14P_1'' + 11P_2'' \Rightarrow P_1'' < \frac{1}{2}P_2''$ (1) (This condition is sufficient for WARP to hold)

For SARP to hold:

$8P_1'' + 20P_2'' > 14P_1'' + 11P_2'' \Rightarrow P_1'' < \frac{3}{2}P_2''$ (2) — Note that (1) includes (2). So,

C is not revealed preferred to A .

Thus, when C is chosen, A should be out of budget set.

the necessary condition for SARP to hold is $P_1'' < \frac{1}{2}P_2''$

④ (a) MRS is given by $\frac{C+1}{F}$. Thus, tangency is $\frac{C+1}{F} = \frac{1}{4}$, which implies $F = 4C+4$.

Plugging this into the budget line, $4C + F = 20$, then solving for C , we get $C^* = 2$, and then $F^* = 12$, an interior solution.

(b) Now, tangency implies $\frac{C+1}{F} = \frac{4}{4} = 1$, that is $F = C+1$. Plugging this into the budget line, $4C + 4F = 20$, then solving for C , we get $C^{**} = 2$, then $F^{**} = 3$, an interior solution.

(c) The total effect is $TE = 3 - 12 = -9$. To find substitution effect, keep purchasing power constant so that Carina can just afford the original bundle at the new prices.

At the new prices, the original bundle costs $4 \cdot C^* + 3 \cdot F^* = 4(2+12) = 56$. Thus, we need to compensate Carina with additional \$36. Now, we solve for the bundle she demands at the new prices after the compensation. The tangency at the new prices, that is, $\frac{C+1}{F} = 1$ implies $F = C+1$. Then, the new compensated budget is

$4F + 4C = 56$ or $F + C = 14$. Plugging $F = C+1$ into this new budget we get $2C+1=14$, that is $C = 6.5$ and then $F = 7.5$. Thus, from original food consumption level (12) to this intermediate level (7.5) is the substitution effect:

$$SE = F - F^* = 7.5 - 12 = -4.5$$

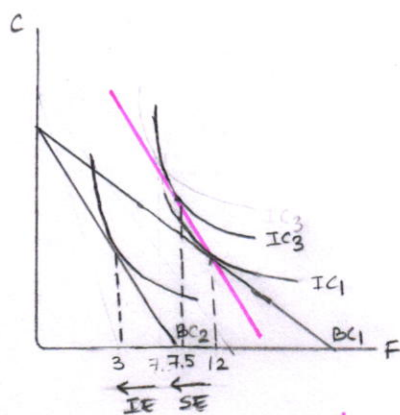
And the income effect is the rest, from this intermediate level (7.5) to the final level (3): $IE = F^{**} - F = 3 - 7.5 = -4.5$. Note that $SE + IE = -4.5 - 4.5 = -9 = TE$ and that $TE = F^{**} - F^* = 3 - 12 = -9$.

Note: Slutsky method
• Isolate the substitution effect by adjusting the consumer's money income so that s/he can just afford the original bundle!

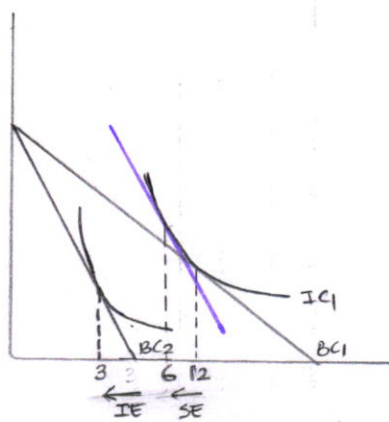
(d) The total effect is $TE = 3 - 12 = -9$. To find substitution effect, keep purchasing power constant so that Carina can just achieve the original utility level. The original utility level is $U = FC + F = 12 \cdot 2 + 12 = 36$. The tangency at the new prices, that is, $\frac{C+1}{F} = 1$ implies that $36 = (C+1)C + C+1 = C^2 + 2C+1 = (C+1)^2$, that is, $C+1=6$, hence $C=5$ and $F=6$.

Thus, from original food consumption level (12) to this intermediate level (6) is the substitution effect: $SE = F - F^{**} = 6 - 12 = -6$. And the income effect is the rest, from the intermediate level (6) to the final level (3): $IE = F^{**} - F = 3 - 6 = -3$. Note that $SE + IE = TE = 6 - 3 = -9$ and that $TE = F^{**} - F^* = 3 - 12 = -9$.

Note: Hicksian method
• Isolate the SE $\Rightarrow$ what would the consumer's optimal bundle be if s/he faced the new lower price for X_1 but experienced no Δ in real income.
— return the consumer to the original IC.



• Slutsky compensation



• Hicksian compensation

(e) $u(F, C) = 2F^{1/2} + C$

With the new utility function, MRS is given by $\frac{1}{F^{1/2}}$, which is diminishing. The tangency gives $\frac{1}{F^{1/2}} = \frac{1}{4}$, that is $F^* = 16$. Then, from the budget line $C^* = (20 - F)/4 = 1$. Thus, the original bundle is $(F^* = 16, C^* = 1)$, and the utility is $u = 2 \cdot 16^{1/2} + 1 = 9$.

After the price change, the tangency implies $\frac{1}{F^{1/2}} = 1$, that is, $F^{**} = 1$. Then the budget line at the new prices implies that $4 \cdot 1 + 4 \cdot C = 20$, that is, $C^{**} = 4$. So, the new bundle is $(F^{**} = 1, C^{**} = 4)$. The total effect in F is $TE = 1 - 16 = -15$.

Slutsky compensation: To keep purchasing power the same so that she can just afford the original bundle, need $4C^* + 4F^* = 4(16 + 1) = 68$. After the compensation so that the income is 68, we get, at the new prices together with the tangency that $\frac{1}{F^{1/2}} = 1$.

Thus, F is still 1, and C satisfies $68 = 4(1 + C)$, that is, $C = 16$.

The intermediate bundle is $(F = 1, C = 16)$. The substitution effect is $SE = F - F^* = 1 - 16 = -15 (TE)$.

There is no income effect: $IE = F^{**} - F = 1 - 1 = 0$.

Hicksian compensation: To keep purchasing power the same so that she can just achieve the original utility level 9, we need $2F^{1/2} + C = 9$. Together with the tangency at the new prices we have $\frac{1}{F^{1/2}} = 1$. Thus, F is still 1, and C satisfies $9 = 2 \cdot 1 + C$, that is $C = 7$.

The intermediate bundle is $(F = 1, C = 7)$. The substitution effect is $SE = F - F^{**} = 1 - 16 = -15 = TE$.

And there is no income effect: $IE = F^{**} - F = 1 - 1 = 0$.

5) (a) $u(x_1, x_2) = x_1^2 x_2$

$I = \$600, p_2 = \$1, p_1 = \$10 \rightarrow p_1' = \5

The utility function is equivalent to $u(x_1, x_2) = x_1^{2/3} x_2^{2/3}$. Then the optimal bundle equation is $x_1^* = \frac{2}{3} \frac{I}{p_1}, x_2^* = \frac{1}{3} \frac{I}{p_2}$. Therefore, initial bundle is $x_1^* = 40, x_2^* = 200$. Final bundle is

$x_1^{**} = 80, x_2^{**} = 200$. The total change in x_1 is $TE = x_1^{**} - x_1^* = 80 - 40 = 40$.

(b) Keep purchasing power constant so that the original utility level is just attainable. The original utility level is $u = 40^2 \cdot 200 = 32 \cdot 10^4$. The tangency at the new prices implies

$\frac{2x_1 x_2}{x_1^2} = \frac{2x_2}{x_1} = 5$, that is, $x_2 = (5/2) x_1$. This together with $u = 32 \cdot 10^4 = x_1^2 x_2$, imply

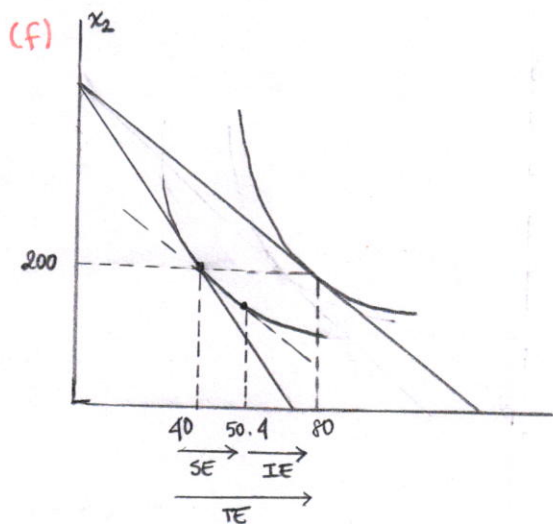
that $x_1 = 40 \sqrt[3]{2}$, therefore $SE_1 = x_1 - x_1^* = 40 \sqrt[3]{2} - 40 = 10.4$. Rest is income effect:

$IE_1 = x_1^{**} - x_1 = 80 - 40 \sqrt[3]{2} = 29.6$.

(c) Keep purchasing power constant so that the original bundle ($x_1^* = 40, x_2^* = 200$) is just affordable at the new prices $p_1' = 5, p_2 = 1$, that is, the compensated income must be 5. $40 + 1 \cdot 200 = 400$. At the new prices with this compensated income the bundle demanded would be $x_1 = (2/3)(400/5) = 160/3$ and $x_2 = (1/3)(400/1) = 400/3$. Therefore $SE_1 = x_1 - x_1^* = (160/3) - 40 = 13.33$. Rest is income effect: $IE_1 = x_1^{**} - x_1 = 80 - (160/3) = 26.67$.

(d) It's an ordinary good since as p_1 decreases, consumption of good 1 increases.

(e) when p_1 decreases, real income increases. Since $IE > 0$, Good 1 is a normal good.



- ⑥ $u(x_1, x_2) = 4 \ln x_1 + \ln x_2 \Rightarrow$ take monotonic transformation
 $\dagger$ work with $u(x_1, x_2) = x_1^4 x_2$

(a) with initial prices $p_1=10$ and $p_2=1$, $m=100$, we get demand

$$x_1(p_1, p_2, m) = \frac{4m}{5p_1} = 8 \text{ and } x_2(p_1, p_2, m) = \frac{m}{5p_2} = 20.$$

With new price $p_1' = 5$ we have that

$$x_1(p_1', p_2, m) = \frac{4m}{5p_1'} = 16 \text{ and } x_2(p_1', p_2, m) = \frac{m}{5p_2} = 20.$$

So, the change in the number of movies (x_1) watched is:

$$\Delta x_1 = x_1(p_1', p_2, m) - x_1(p_1, p_2, m) = 16 - 8 = 8.$$

(b) Movies are ordinary goods since the number demanded increases when its price decreased (demand is downward sloping).

(c) To find the substitution effect, we first need to find m' , which is the amount of income that would be required to purchase the $(x_1, x_2) = (8, 20)$ bundle, which was optimal at the original prices) at the new price p_1' :

$$m' = p_1' \cdot x_1 + p_2 x_2 = 5 \cdot 8 + 1 \cdot 20 = 60.$$

So, to purchase the original bundle at the new prices would require an income of $m' = \$60$ (rather than \$100). To find what amount of the total change can be attributed to the substitution effect, we need to find the quantity demanded with income $m' = 60$ and price $p_1' = 5$:

$$x_1(p_1', p_2, m') = \frac{4m'}{5p_1'} = 9 \frac{3}{5}.$$

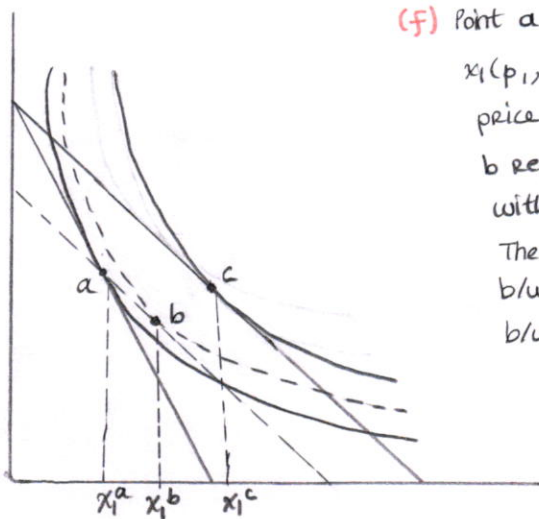
So, we have that the substitution effect is:

$$\Delta x_1^S = x_1(p_1', p_2, m) - x_1(p_1', p_2, m') = 16 - 9 \frac{3}{5} = 6 \frac{7}{5}.$$

(d) The income effect is

$$\Delta x_1^N = x_1(p_1', p_2, m) - x_1(p_1', p_2, m') = 16 - 9 \frac{3}{5} = 6 \frac{7}{5}.$$

(e) The income effect is positive (with prices p_1' and p_2 , less of x_1 was demanded with the lower hypothetical m' than was demanded at income m .) This means preferences over these goods are normal.



(f) Point a represents the initial bundle, where demand for $x_1(p_1, p_2, m) = 8$. Point c represents demand after the price change where we had $x_1(p_1', p_2, m) = 16$. Point b represents the optimal (and hypothetical) value with p_1' and m' , with x_1 demand $x_1(p_1', p_2, m') = 9 \frac{3}{5}$. The difference b/w a and c is the total effect; b/w a and b is the substitution effect and b/w b & c is the income effect.

⑦ Recall that Trevor's preferences are represented by the utility function $u(x_1, x_2) = \min\{5x_1, x_2\}$. (He consumes five times as many as strawberries (x_2) as he does milk (x_1) in making the perfect strawberry milkshake). We found that his demand functions are given by:

$$x_1 = \frac{m}{p_1 + 5p_2} \quad \text{and} \quad x_2 = \frac{5m}{p_1 + 5p_2}$$

(a) with $m=200$, $p_1=15$, and $p_2=1$, his demands for milk (x_1) and strawberries (x_2) are:

$$x_1(p_1, p_2, m) = \frac{m}{p_1 + 5p_2} = \frac{200}{15 + 5 \cdot 1} = 10 \quad \text{and} \quad x_2(p_1, p_2, m) = \frac{5m}{p_1 + 5p_2} = \frac{5 \cdot 200}{15 + 5 \cdot 1} = 50.$$

with the price of milk decreasing to $p_1'=5$, his new quantities demanded become:

$$x_1(p_1', p_2, m) = \frac{m}{p_1' + 5p_2} = \frac{200}{5 + 5 \cdot 1} = 20 \quad \text{and} \quad x_2(p_1', p_2, m) = \frac{5m}{p_1' + 5p_2} = \frac{5 \cdot 200}{5 + 5 \cdot 1} = 100.$$

The total change in the demand for milk is:

$$\Delta x_1 = x_1(p_1', p_2, m) - x_1(p_1, p_2, m) = 20 - 10 = 10.$$

(b) The substitution effect, Δx_1^S , for complementary goods is always zero. Let's verify this. First, we need to find the m' income that would make the original bundle just affordable with the price change:

$$m' = p_1' x_1 + p_2 x_2 = 5 \cdot 10 + 1 \cdot 50 = 100$$

Next, we find the quantity of x_1 demanded with $m'=100$, $p_1'=5$ and $p_2=1$:

$$x_1(p_1', p_2, m') = \frac{m'}{p_1' + 5p_2} = \frac{100}{5 + 5 \cdot 1} = 10$$

So, we have that the substitution effect is:

$$\Delta x_1^S = x_1(p_1', p_2, m') - x_1(p_1, p_2, m) = 10 - 10 = 0.$$

(c) The income effect is

$$\Delta x_1^N = x_1(p_1', p_2, m) - x_1(p_1', p_2, m') = 20 - 10 = 10$$

(since $\Delta x_1 = \Delta x_1^S + \Delta x_1^N$ we can get this from $10 = 0 + \Delta x_1^N \Rightarrow \Delta x_1^N = 10$ as well).

⑧ (a) Since $\frac{MU_1}{P_1} = \frac{2}{1} > \frac{1}{1} = \frac{MU_2}{P_2}$, we must have a corner solution with all spent on x_1 .

Thus, we have $x_1^* = 5/1 = 5$, and $x_2^* = 0$.

(b) Since now $\frac{MU_1}{P_1} = \frac{2}{3} < \frac{1}{1} = \frac{MU_2}{P_2}$, we must have a corner solution with all spent

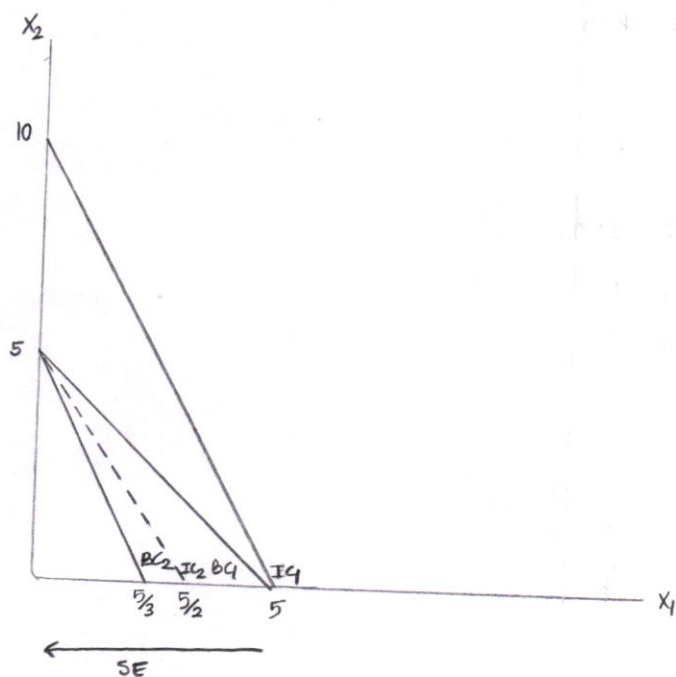
on x_2 . Thus, we have $x_1^{**} = 0$ and $x_2^{**} = 5$.

(c) To get the same utility with the initial bundle with new price level, we need $x_1 = 0$ and $x_2 = 10$. So, $TE = SE = -5$, and $IE = 0$.

(d) Now, $\frac{MU_1}{P_1} = \frac{2}{1.25} > \frac{1}{1} = \frac{MU_2}{P_2}$, we must have a corner solution spent on x_1 .

Thus, we have $x_1^{**} = 4$, $x_2^{**} = 0$. Total change is now $TE = -1$. Similarly, to get the same utility with the new price level, we must have $x_1 = 5$, $x_2 = 0$. There is no substitution effect, $SE = 0$ and all of the total effect is due to income effect.

$TE = IE = -1$.



Intermediate x_1 level = 0 } $SE = 0 - 5 = -5$
 First x_1 level = 5
 Last x_1 level = 0 $IE = 0 - 0 = 0$

⑨ (a) Derive the Hicksian demand functions, $h_1(p_1, p_2, \bar{u})$ and $h_2(p_1, p_2, \bar{u})$, assuming interior solutions:

The Hicksian demands (if both are positive) are the solution of the tangency condition,

$$\frac{1}{x_1} = \frac{p_1}{p_2} \Rightarrow h_1(p_1, p_2, \bar{u}) = \frac{p_2}{p_1} \quad \text{and} \quad h_2(p_1, p_2, \bar{u}) = \bar{u} - \ln(p_2/p_1)$$

(b) The initial demands are:

$$x_1^*(p_1, p_2, m) = \frac{p_2}{p_1} = 3 \quad \text{and} \quad x_2^*(p_1, p_2, m) = \frac{m}{p_2} - 1 = 5$$

The corresponding utility level is: $\bar{u} = \ln(3) + 5$

We therefore obtain the Hicksian demands at the new prices with original utility level:

$$h_1(p_1', p_2, \bar{u}) = \frac{p_2}{p_1'} = 2 \quad \text{and} \quad h_2(p_1', p_2, \bar{u}) = \bar{u} - \ln p_2/p_1' = \ln(3) + 5 - \ln(2) = \ln(3/2) + 5$$

• The substitution effects are:

$$SE_1 = h_1(p_1', p_2, \bar{u}) - x_1^*(p_1, p_2, m) = 2 - 3 = -1$$

$$SE_2 = h_2(p_1', p_2, \bar{u}) - x_2^*(p_1, p_2, m) = \ln(3/2)$$

• The new (final) demand levels at the new prices are:

$$x_1^*(p_1', p_2, m) = \frac{p_2}{p_1'} = 2 \quad \text{and} \quad x_2^*(p_1', p_2, m) = \frac{m}{p_2'} - 1 = 5$$

so, the income effects are:

$$IE_1 = x_1^*(p_1', p_2, m) - h_1(p_1', p_2, \bar{u}) = 2 - 2 = 0$$

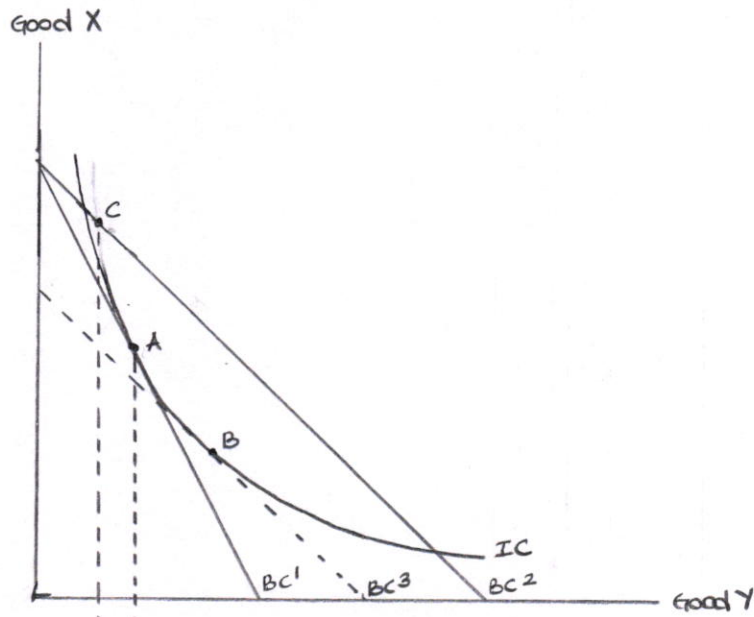
$$IE_2 = x_2^*(p_1', p_2, m) - h_2(p_1', p_2, \bar{u}) = -\ln(3/2)$$

And the total effects are:

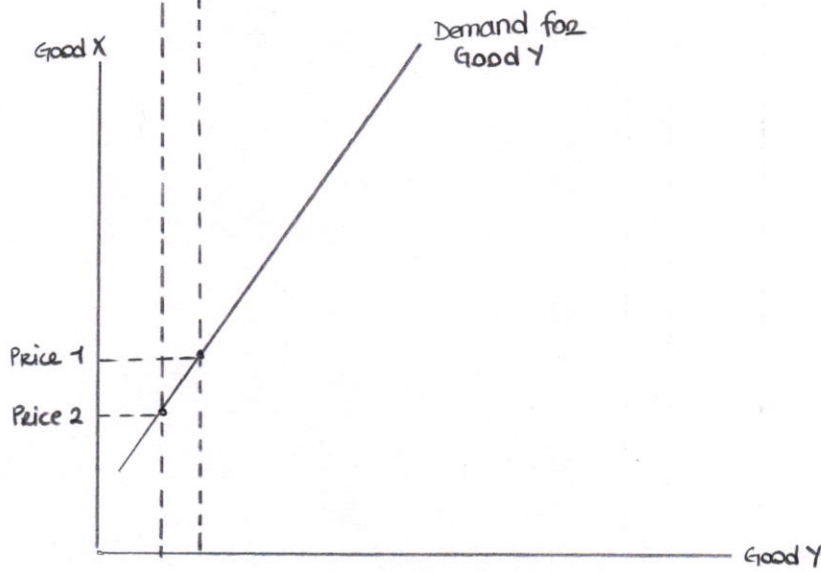
$$TE_1 = x_1^*(p_1', p_2, m) - x_1^*(p_1, p_2, m) = 2 - 3 = -1$$

$$TE_2 = x_2^*(p_1', p_2, m) - x_2^*(p_1, p_2, m) = 5 - 5 = 0.$$

(10)



- Giffen Good - consume more of as the price rises or vice versa.



② (a) FALSE.

Consumer surplus CS is an exact measure of a change in consumer welfare if and only if it equals both equivalent and compensating variation, that is, $CS = EV = CV$. CS is measured as the area beneath the Marshallian demand curve between any two prices. EV and CV are measured as the area beneath the Hicksian demand curves between those same two prices. For a given price change, these areas are the same if and only if the Marshallian and Hicksian demand curves are the same. It is enough to show that these two demand curves are not the same when the substitution effect is zero. There are two ways to do this.

First Approach

- Illustrate your argument with a diagram. Doing so requires you to make appropriate assumptions about the shape of the indifference curves. We want $SE=0$. So assume the consumer must consume goods in fixed proportion, i.e. perfect complements. Then, for a given price change, construct both demand curves to show that they are not the same, and thus the area beneath these two curves cannot be the same either.

Second Approach

- Use the Slutsky equation, which provides a precise statement of our graphical analysis. The Slutsky eqn. is given as:

$$\frac{\partial d_i}{\partial p_i} = \frac{\partial h_i}{\partial p_i} - \frac{\partial d_i}{\partial M} q_i.$$

where d_i is the Marshallian demand for good i , h_i is the Hicksian demand for good i , M is money income, and q_i is the initial quantity of good i . It says that the difference b/w the Marshallian demand response to a price change ($\partial d_i / \partial p_i$ - which measures the change in CS) and the Hicksian demand response to a price change ($\partial h_i / \partial p_i$ - which measures CV and EV) is equal to the income effect scaled by the effective change in income due to the price change (recalling that $q_i = \partial E / \partial p_i$).

We can therefore conclude that $CS = CV = EV$ if and only if the income effect is equal to zero, i.e., $\frac{\partial d_i}{\partial M} q_i$ - which in economic terms can happen if the good in question is neither inferior or normal (i.e., $\frac{\partial d_i}{\partial M} = 0$) or if the consumer is consuming $q_i = 0$ initially.

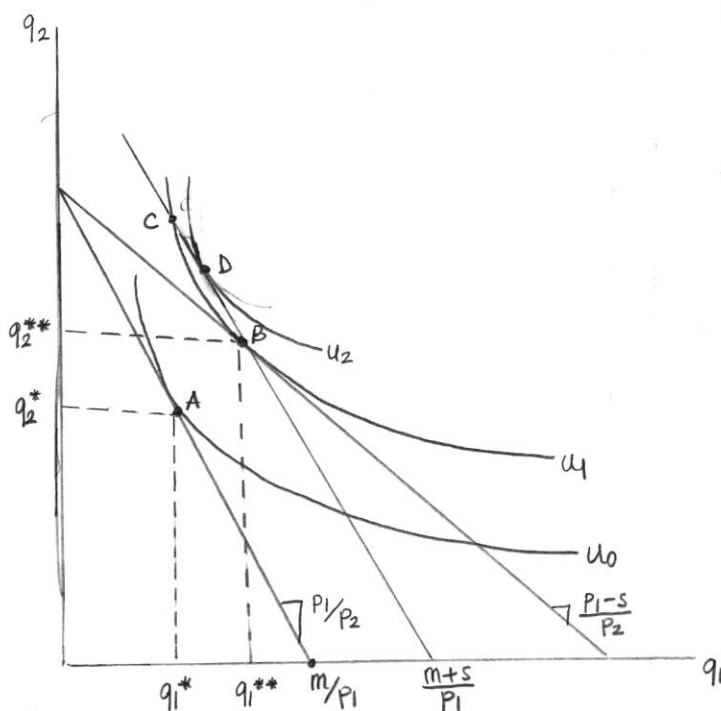
(b) FALSE.

other way around. let the initial quantity demanded of each good in the absence of a subsidy be given by the bundle $A = (q_1^*, q_2^*)$ on indifference curve u_0 , where $MRS(q_1^*, q_2^*) = P_1/P_2$ and the budget constraint holds.

- A subsidy is applied to p_1 decreases the relative price of good 1 to $\frac{P_1 - S}{P_2}$. This gives the consumer an incentive to substitute toward good 1. let the new quantities (with the subsidy) be given by bundle $B = (q_1^{**}, q_2^{**})$ on indifference curve u_1 , where $q_1^{**} > q_1^*$ and $u_1 > u_0$. At this solution, we have $MRS(q_1^{**}, q_2^{**}) = \frac{P_1 - S}{P_2}$ and the government incurs a cost equal to sq_1^{**} to administer the subsidy program.

- A lump sum cash grant S that is large enough to make bundle B affordable (i.e. shifts the budget constraint out such that it passes through bundle B) will cost the govt. the same amount of money as a subsidy, i.e. $S = sq_1^{**}$, but will make the consumer better off (strictly so, assuming convex preferences). This is because, at bundle B, we now have $MRS(q_1^{**}, q_2^{**}) < P_1/P_2$; i.e. the slope of the indifference curve at bundle B is smaller than the slope of the original budget constraint shifted out by an amount equal to S . In words, this means that the relative price of good 1 exceeds the marginal value of good 1 and thus a consumer with carte blanche as to how to spend subsidy S will substitute away from good 1 (toward good 2) in order to make themselves better off. (i.e., to reach indifference curve $u_2 > u_1$).

- This implies that a lump-sum cash grant need not be as large as S in order to help the consumer reach indifference curve u_1 . Thus, lump-sum cash grants are less expensive to taxpayers.



• with per unit subsidy:

$$\text{utility max} \Rightarrow A = (q_1^*, q_2^*) \rightarrow B = (q_1^{**}, q_2^{**})$$

• with cash to make bundle B feasible:

$$\text{utility max} \Rightarrow A = (q_1^*, q_2^*) \rightarrow D = (\hat{q}_1, \hat{q}_2)$$

$$\text{where } u_2 > u_1 > u_0$$

(c) FALSE.

Again, we could use the Slutsky eqn to answer this question:

$$\frac{\partial d_i}{\partial p_i} = \frac{\partial h_i}{\partial p_i} - \frac{\partial d_i}{\partial m} q_i$$

where d_i is the Marshallian demand for good i , h_i is the Hicksian demand for good i , m is the money income, and q_i is the initial quantity of good i . The first two terms are the slopes of the Marshallian and Hicksian demand curves, respectively.

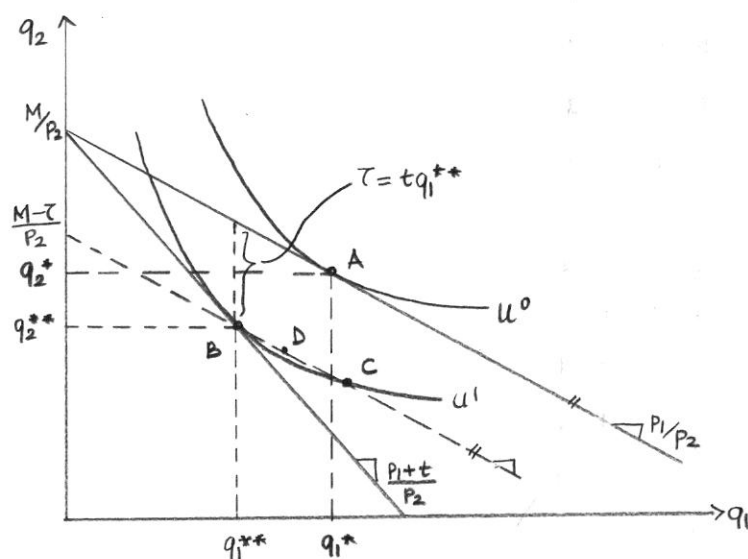
As noted above, it is sufficient to show that these two slopes are the same if the income effect is zero, i.e., if the product $\partial d_i / \partial m \cdot q_i = 0$. Of course, one could also argue that the Marshallian demand curve will be steeper than the Hicksian demand curve as long as good i is inferior, i.e., $\partial d_i / \partial m < 0$. In other words, the Total Price Effect from a given price change $\partial d_i / \partial p_i$ is smaller than the pure SE $\partial h_i / \partial p_i$ if the IE $\partial d_i / \partial m$ is negative and countervailing.

12. Let the initial quantity demanded of each good in the absence of a tax be given by the bundle $A = (q_1^*, q_2^*)$, where $MRS(q_1^*, q_2^*) = \frac{P_1}{P_2}$ and the budget constraint holds.

• An excise tax t imposed on p_1 increases the relative price of good 1 to $\frac{P_1+t}{P_2}$. This gives the consumer an incentive to substitute out of good 1. Let the new quantities (with the excise tax) be given by bundle $B = (q_1^{**}, q_2^{**})$ where $q_1^{**} < q_1^*$. At this solution, we have $MRS(q_1^{**}, q_2^{**}) = \frac{P_1+t}{P_2}$ and the government raises an amount of tax revenue equal to tq_1^{**} .

• A lump sum tax τ must raise the same amount of tax revenue, i.e., $\tau = tq_1^{**}$. This means bundle B must still be affordable with the lump-sum tax, i.e. the budget constraint shifts inward through bundle B . But, at bundle B , we now have $MRS(q_1^{**}, q_2^{**}) > P_1/P_2$; that is, the slope of the indifference curve at bundle B is larger than the slope of the original budget constraint shifted in by an amount equal to τ . In words, this means that the marginal value of good 1 exceeds its relative price and thus the consumer will substitute toward good 1 to make themselves better off.

• Thus, lump-sum taxes are always (weakly) preferred to excise taxes of the same size because consumers have carte blanche as how to spend any remaining income and can therefore make any substitutions necessary to make themselves better off. (i.e., reach a higher indifference curve)



- (i)
 - start at bundle $A = (q_1^*, q_2^*)$
 - Excise tax $\Rightarrow$ New optimum at bundle $B = (q_1^{**}, q_2^{**})$
 - Tax revenue raised equals per unit tax, t , multiplied by new quantity of good 1, i.e., tq_1^{**}
- (ii) Lump-sum taxes must raise the same amount so that $tq_1^{**} = \tau$ means bundle $B = (q_1^{**}, q_2^{**})$ must still be affordable with lump sum tax.
- (iii) Given $\tau = tq_1^{**}$, substitution to any bundle (such as D) on line segment $\overline{BC}$ is possible.
 - Since bundle $D > B$, lump-sum taxes are preferable.