

⑦ (a) Arbitrarily pick two bundles $x=(x_1, x_2)$ and $y=(y_1, y_2)$ and show that both u and v represent the same preference relation. Assume that $x \succ y$, then, $u(x_1, x_2) > u(y_1, y_2)$, then $x_1 + x_2 > y_1 + y_2$, then $\log x_1 + \log x_2 > \log y_1 + \log y_2$, that is, $v(x_1, x_2) > v(y_1, y_2)$.

(b) Consider $(1,1)$ and $(0,2)$. According to u , $u(1,1)=2$; $u(0,2)=2$, then $(1,1) \sim (0,2)$. According to z , $z(1,1)=2$, $z(0,2)=4$, then $(1,1) \prec (0,2)$. So, they are not the same preferences, hence z does not represent the same preferences as u does.

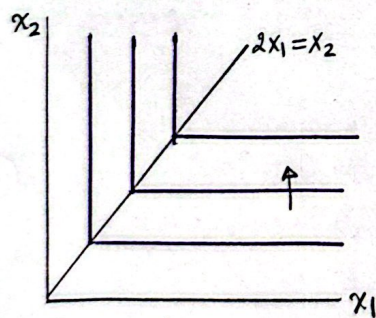
⑧ All these three utility functions have the same MRS_{xy} , since all utility functions are monotonic transformation of each other.

For $u(x,y)=xy$; $mu_x=y$, $mu_y=x$ and $MRS_{xy}=y/x$

For $u(x,y)=x^2y^2$; $mu_x=x^2y^2$, $mu_y=2x^2/y \Rightarrow MRS_{xy}=y/x$

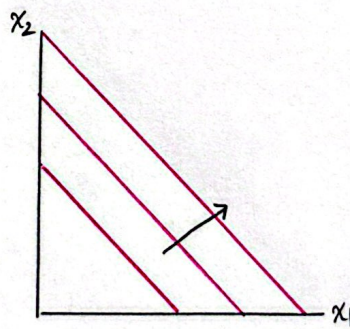
For $u(x,y)=\ln x + \ln y$; $mu_x=1/x$, $mu_y=1/y$ and $MRS_{xy}=y/x$

⑨ (a)



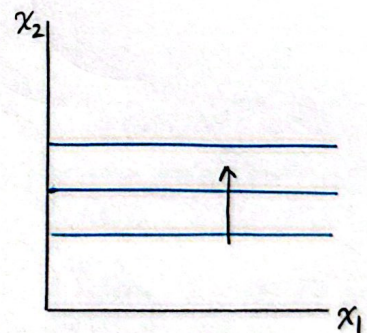
(i) $u(x_1, x_2) = \min\{2x_1, x_2\}$

• Nazlı's preferences are perfect complements



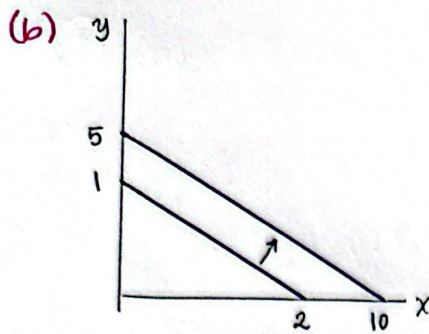
(ii) $u(x_1, x_2) = x_1 + x_2$

• Betül's preferences are perfect substitutes



(iii) $u(x_1, x_2) = x_2$

• x_1 is a neutral good and x_2 is preferred good according to Tekin's preferences.

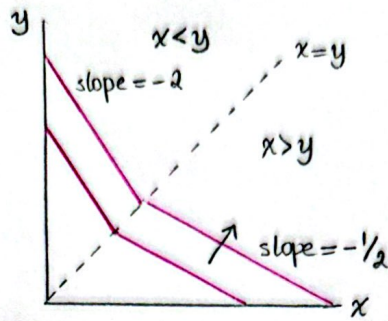


(i) $u(x, y) = x + 2y$

$u = 10 = x + 2y$

$x = 0 \Rightarrow y = 5$

$y = 0 \Rightarrow x = 10$

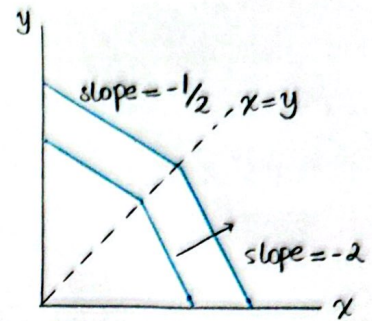


(ii) $u(x, y) = \min \{ 2x + y, x + 2y \}$

$2x + y = x + 2y \Rightarrow x = y$ (kink)

$x > y \Rightarrow u(x, y) = x + 2y$

$x < y \Rightarrow u(x, y) = 2x + y$

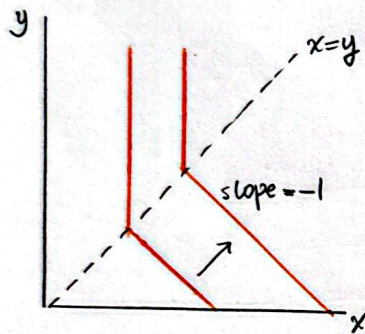


(iii) $u(x, y) = \max \{ 2x + y, x + 2y \}$

$2x + y = x + 2y \Rightarrow x = y$

$x > y \Rightarrow u(x, y) = 2x + y$

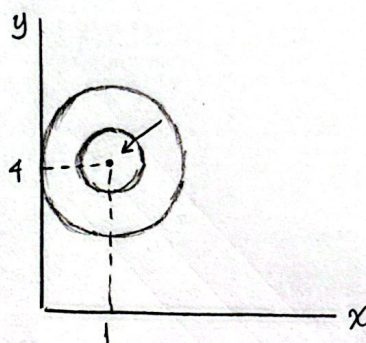
$x < y \Rightarrow u(x, y) = x + 2y$



(iv) $u(x, y) = x + \min \{ x, y \}$

$x > y \Rightarrow u(x, y) = x + y$

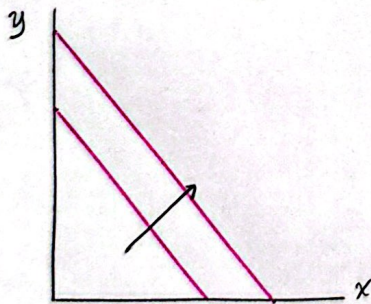
$x < y \Rightarrow u(x, y) = 2x$



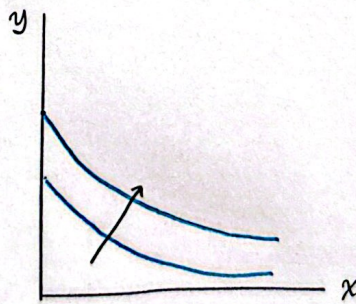
(v) $u(x, y) = -(x-1)^2 - (y-4)^2$

• classical equation of a circle centered (1, 4) and (1, 4) is the satiation point.

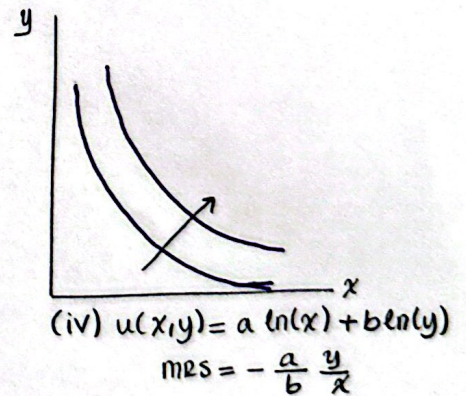
(c)



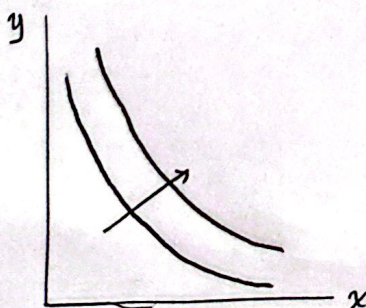
(ii) $u(x, y) = ax + by$
 $mps = -a/b$



(iii) $u(x, y) = \ln x + y$
 $mps = -1/x$



(iv) $u(x, y) = a \ln(x) + b \ln(y)$
 $mps = -\frac{a}{b} \cdot \frac{y}{x}$



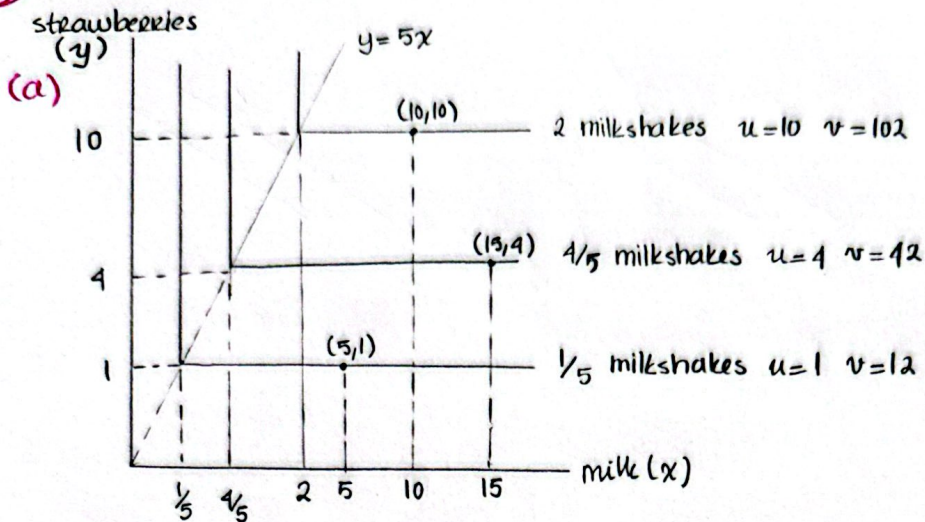
(i) $u(x, y) = x^\beta y^{1-\beta}$ where $0 < \beta < 1$

$mps = -\frac{mu_1}{mu_2} = -\frac{\partial u / \partial x_1}{\partial u / \partial x_2}$

$= -\frac{\beta \cdot x^{\beta-1} y^{1-\beta}}{(1-\beta) x^\beta y^{-\beta}} = -\frac{\beta}{1-\beta} \cdot \frac{y}{x}$

(5)

10

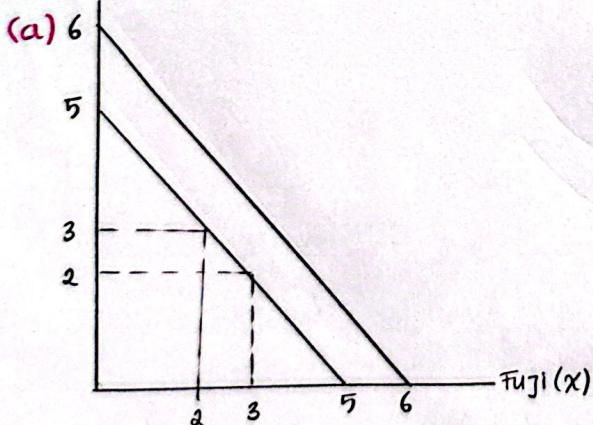


• MRS at each of these bundles is zero!

(b) A utility function that represents that preferences would be $u(x, y) = \min \{5x, y\}$.
The utility levels would be $u(5, 1) = 1$, $u(10, 10) = 10$ and $u(15, 4) = 4$.

(c) we have, $v(x, y) = 10 \cdot \min \{5x, y\} + 2$. Since v is a monotonic transformation of u , shape of the indifference curve and preference rankings will not be affordable. But, the numbers utility levels assigned on each indifference curve will change. We have:
 $v(5, 1) = 12$; $v(10, 10) = 102$; and $v(15, 4) = 42$.

11



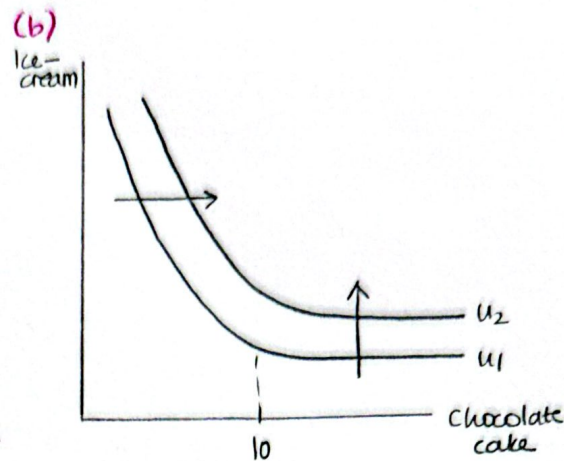
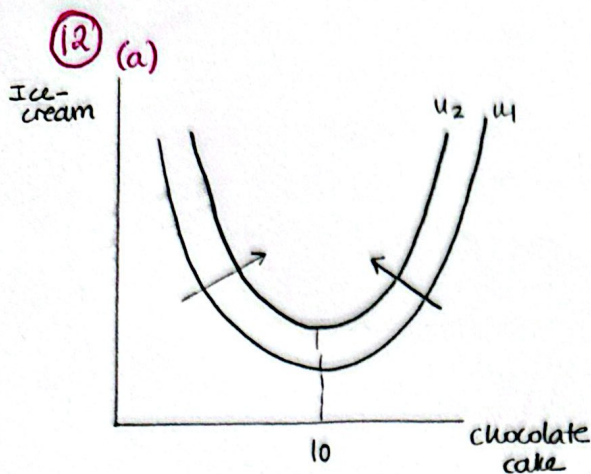
(b) Here are some examples:

$$u(x, y) = x + y, \quad u(x, y) = (x + y)^2,$$

$$u(x, y) = 5(x + y), \quad u(x, y) = x + y + 100$$

(c) MRS is negative of the slope of IC which is -1 in this case.

6



(13) $u(x_g, x_b) = x_g + \sqrt{x_b}$

(a) $\bar{u} = x_g + \sqrt{x_b} \Rightarrow x_g = \bar{u} - \sqrt{x_b}$
 $= \bar{u} - x_b^{1/2}$

(b)
$$\left. \begin{aligned} m_{ub} &= \frac{1}{2} (x_b)^{-1/2} \\ m_{ug} &= 1 \end{aligned} \right\} MRS_{bg} = \frac{m_{ub}}{m_{ug}} = \frac{1}{2} x_b^{-1/2}$$

(c) Convexity of preferences is equivalent to the diminishing MRS property. You can check whether MRS is decreasing, as x_b increases and x_g decreases. MRS is actually decreasing since $\frac{1}{2} x_b^{-1/2}$ is decreasing in x_b . Alternatively, you can check the second order derivative of the curve you found: $\frac{dx_g}{dx_b} = -\frac{1}{2} x_b^{-1/2}$, then $\frac{d^2 x_g}{d^2 x_b} = \frac{1}{4} x_b^{-3/4}$. Second order derivative is positive, so the indifference curve has a convex slope.

(d) Tangency is equivalent to $MRS_{bg} = \frac{P_b}{P_g}$. Then, $\frac{1}{2} x_b^{-1/2} = \frac{P_b}{P_g}$, that is, $x_b^{1/2} = \frac{P_g}{2P_b}$ which implies, $x_b = \frac{1}{4} \left(\frac{P_g}{P_b} \right)^2$.

(e) $P_g = \$2$, $P_b = \$1$, $m = \$8$

Since this utility function is well-behaved (it is monotone, represents convex preferences since MRS is diminishing, and continuous & differentiable) at an interior solution tangency should hold. From the tangency condition above, plugging prices, we get:

$x_b^* = \frac{1}{4} \cdot \left(\frac{2}{1} \right)^2 = \frac{1}{4} \cdot 4 = 1$. Then, from the budget equation, we get $1 \cdot 1 + 2 \cdot x_g = 8$, that is, $x_g^* = 7/2$. Since both $x_b^* = 1$ and $x_g^* = 7/2$ are positive, we have an interior solution at $(x_b^* = 1, x_g^* = 7/2)$.

(7)

(f) From the tangency condition, we get $x_b = 25/4$. However, this together with budget equation implies a negative amount of guns. Hence, the best solution is to buy no guns and buy as much as bangos as possible. Mark spends all income on bangos: $x_b^* = \frac{m}{p_b} = 4$, and none spent on guns, $x_g^* = 0$.

You can also see this through comparing $\frac{mub}{p_b}$ and $\frac{mug}{p_g}$. We have $mub = \frac{1}{2} x_b^{-1/2}$

and $mug = 1$; that is, $\frac{mub}{p_b} = \frac{\frac{1}{2} x_b^{-1/2}}{1} = \frac{1/2}{x_b^{1/2}}$ and $\frac{mug}{p_g} = \frac{1}{5}$. When spending all income

on bangos, the most he can buy is $x_b = 4$. With this amount $\frac{mub}{p_b} = \frac{1/2}{4^{1/2}} = 1/4$

which is larger than $1/5$. Thus, we would like to get more x_b , but $x_b = 4$

is the limit. Thus, this is the solution, $x_b^* = 4$, together with no guns, $x_g^* = 0$.

⑭ (a) $u(x_1, x_2) = x_1^a x_2^{1-a}$

This is a Cobb-Douglas utility function, which is well-behaved (it is increasing in x and y , that is, it is monotone. It has DMRS, that is, it represents convex preferences. It is also continuous and differentiable. You should be able to check monotonicity and DMRS). And the budget line is $MRS = \frac{mu_1}{mu_2} = \frac{a \cdot x_1^{a-1} x_2^{1-a}}{(1-a) x_1^a x_2^{-a}} = \frac{a}{1-a} \cdot \frac{x_2}{x_1}$.

And the price ratio is simply p_1/p_2 . Thus, we get:

$$\frac{a}{1-a} \cdot \frac{x_2}{x_1} = \frac{p_1}{p_2}$$

Then, we have $x_2 = x_1 \cdot \frac{1-a}{a} \cdot \frac{p_1}{p_2}$. Now, plug this into the budget line and solve

for x_1 .

$$p_1 x_1 + p_2 x_1 \cdot \frac{1-a}{a} \cdot \frac{p_1}{p_2} = m$$

that is, $p_1 x_1 + x_1 \cdot \frac{1-a}{a} p_1 = m$, which implies $x_1 \left[p_1 + \frac{1-a}{a} p_1 \right] = m$ or $x_1 p_1 \left[1 + \frac{1-a}{a} \right] = m$,

that is, $x_1 p_1 \cdot \frac{1}{a} = m$. Thus, we get $x_1^* = a \cdot \frac{m}{p_1}$

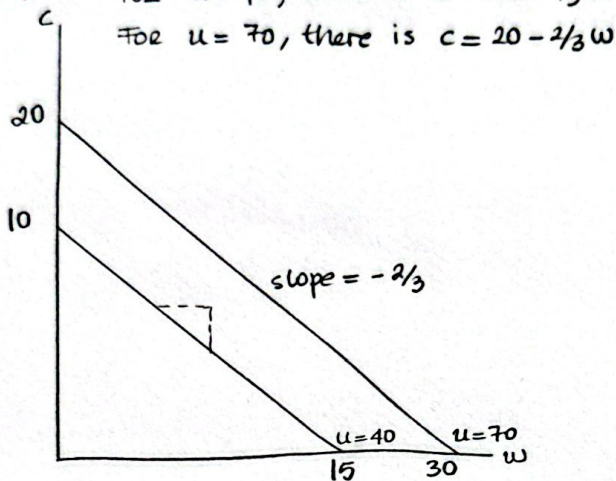
plugging this into the eqn: $x_2 = x_1 \cdot \frac{1-a}{a} \cdot \frac{p_1}{p_2}$, we get $x_2 = a \cdot \frac{m}{p_1} \cdot \frac{1-a}{a} \cdot \frac{p_1}{p_2}$, that is,

$$x_2^* = (1-a) \frac{m}{p_2}$$

(b) The income share is spending on good 1 is $\frac{p_1 x_1^*}{m} = a$, and the income share she is spending on good 2 is:

$$\frac{p_2 x_2^*}{m} = (1-a)$$

(15) (a)



(b) Let $u = \bar{u}$, we have $c = \frac{\bar{u}-10}{3} - \frac{2}{3}w$,
 then $MRS = -\frac{dc}{dw} = \frac{2}{3}$

(c) If the consumption of music increases to 20, it lifts the linear indifference curve inward toward the origin. The wine intercept is reduced by 5 units for both indifference curves. This is because we need less wine and cheese to achieve the same utility levels. Slopes of IC's will not be affected since the rate of substitution of wine for cheese will not be affected by an increase in music consumption.