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ECON 201 - Problem Set 8

1. Draw the isoquants for the following production functions:

(a) $f(x, y) = \max\{2x, y^3\}$

(b) $f(x, y) = 3x - \min\{2x, y\}$

(c) $f(x, y) = (\max\{x, y\})^2 - (\min\{x, y\})^2$

2. For each of the following production functions, (1) find MP_L and MP_K , (2) state whether $MRTS_{L,K}$ is diminishing or not, and (3) state whether it has increasing returns to scale, decreasing returns to scale or constant returns to scale.

(a) $f(L, K) = KL^2 + L$

(b) $f(L, K) = 7L + 12K - 4$

(c) $f(L, K) = 3L^{\frac{1}{3}}K$

(d) $f(L, K) = \min\{3L, K\}$

3. A firm has the production function $f(x_1, x_2) = (x_1^b + x_2^b)^c$ where $b > 0$ and $c > 0$. Find a relation between b and c (such as an inequality) for which this firm will have increasing returns to scale.

4. Suppose a firm's production function is given as:

$$Q(K, L) = 150K^2L^2 - K^3L^3$$

Assume $K = 5$ is fixed.

(a) Derive the firm's total product of labor, marginal product of labor and average product of labor.

(b) Plot MPL and APL on the same graph.

(c) Find the amount of labor at which MPL is maximized.

(d) Find the amount of labor necessary for this production process for $MPL=APL$.

- (e) What is the maximum number of output the firm can produce in the short run?
5. A competitive firm's production function is $f(x, y) = 6\sqrt{x} + 8\sqrt{y}$. The price of factor x is \$1 and the price of factor y is \$4. The price of output is \$8. What is the profit-maximizing quantity of output?
6. Suppose a firm's production function is given as $f(K, L) = \sqrt{8L}$ with input prices r, w and p denotes the output price and assume that K is fixed $K = 1$.
- (a) Write down the profit maximization problem.
- (b) Derive the optimal production plan which maximizes the firm's profit.
- (c) Find the firm's maximized profit level.
- (d) How does the firm's optimal output level change if wage increases or output price decrease?
7. A competitive firm uses a single input x to produce its output y . The firm's production function is given by $y = x^{\frac{3}{2}}$ for quantities of x between 0 and 4. For quantities of x greater than 4, the firm's output is $y = 4 + x$. If the price of the output y is \$1 and the price of the input x is \$3, how much x should the firm use to maximize its profit?
8. Consider the following production functions:

$$f(K, L) = K^2 L^2$$

$$f(K, L) = K + 0.5L^4$$

$$f(K, L) = \sqrt{\min\{K, L\}}$$

- (a) Find the returns to scale for each function.
- (b) Find the cost functions for each of the production functions (Assume $w_L = 1, w_K = 1$).
- (c) Plot the cost functions with y on the horizontal axis and cost on the vertical one.
- (d) Find and plot the average and marginal cost functions with y on the horizontal axis and average cost on the vertical one.
9. Suppose that a firm has the following production function

$$f(x, y) = x^{1/2} + y^{1/2}$$

- (a) Find the unconditional input demand of this firm for each input.
- (b) Find the supply function of the firm.

- (c) Find the profit function.
- (d) Find the conditional input demand of this firm for each input.
- (e) Find the cost function of this firm.

10. A firm uses 4 inputs to produce 1 output. The production function is

$$f(x_1, x_2, x_3, x_4) = \min\{2x_1 + x_2, x_3 + 2x_4\}$$

(a) What is the vector of conditional factor demands to produce 1 unit of output when the factor price vector is $w = (1, 2, 3, 4)$?

(b) What is the cost function when the factor price vector is $w = (w_1, w_2, w_3, w_4)$ for all $w_i > 0, i \in \{1, 2, 3, 4\}$?

(c) What kind of returns to scale does this technology exhibit?

11. A firm has a production function

$$f(x_1, x_2, x_3, x_4) = 2\min\{x_1, x_2\} + 3\min\{x_3, x_4\}$$

If the factor prices $(w_1, w_2, w_3, w_4) = (8, 5, 3, 4)$ for the firm, how much will it cost to produce 70 units of output?

12. Kerem's cupcake factory employs workers (L) and cake-machines (K) according to the following function: $f(K, L) = K^{0.5}L^{0.5}$. The hourly cost of cake machine is \$10 and the hourly cost of workers is \$40.

- (a) Write the Lagrangian for the cost-minimization problem.
- (b) Derive the optimal cake-machine to labour ratio.
- (c) Suppose Kerem wishes to produce 1000 units of cupcakes. How much labour and cake-machine should he employ? How much will it cost to produce?
- (d) An order arrives doubling the amount of cupcake Kerem needs to produce. Assuming he is unable to purchase more machines, how much will it cost to meet the new production level?
- (e) In the long-run, Kerem will be able to employ more machine as well as labour. If he continues to produce 2000 units of cupcakes, how much will it cost in the long run?
- (f) Explain why the cost in the long run is different than in the short run.

13. Suppose production function is given as

$$f(L, K) = \left(\min\left(\frac{K}{a}, \frac{L}{b}\right) \right)^c$$

where $a, b, c > 0$ are all positive constants. The input prices are r and w , for K and L , respectively.

(a) Does this production function exhibit increasing, decreasing or constant returns to scale, when $a = \frac{1}{2}$ and $b = \frac{1}{3}$?

(a) Without specifying a and b , find the conditional input demand functions $K(q, r, w)$, $L(q, r, w)$ and the cost function $c(q, w, r)$.

(b) How are the conditional factor demands $K(q, w, r)$, $L(q, w, r)$ and the cost function $c(q, w, r)$ affected by the prices of the inputs, r and w ? Using economic intuition, what can we say about why this is so?

(c) Let's now fix r and w and rewrite the cost function $c(q, w, r)$ as a function of q only. Assume there is no fixed cost. More specifically, define the constant A such that $A = ra + wb$ and write the cost function as $c(q)$. What is the marginal cost function, $MC(q)$, for this firm? What is the marginal cost function, $ATC(q)$, for this firm? What are the slopes of the respective cost curves $MC(q)$ and $ATC(q)$?

(d) Describe the relationship between returns to scale and the shape of the cost curves $MC(q)$ and $ATC(q)$?

14. Suppose a firm has the following production function, which takes three inputs, L , K and M , and produces a single output: $f(L, K, M) = 3L^{\frac{1}{3}}K^{\frac{1}{3}} + M$. Suppose the input prices are given by w for L , r for K and 1 for M . Find the conditional input demand functions $L(q, w, r)$, $K(q, w, r)$ and $M(q, w, r)$, and the cost function $c(q, w, r)$. Assume interior solutions.

15. For each of the total cost functions given below, (1) find the AC , AVC , AFC and MC , and (2) show that MC intersects the AVC at its minimum. (a) $TC(q) = 2q^2 - q + 4$, (b) $TC(q) = 1000 + 50q^2$, and (c) $TC(q) = q^3 + 1$.