

Dummy Variables

investments & savings

consider:

$$I = \text{investment / gdp}$$

$$S = \text{savings / gdp}$$

theory

$$\begin{array}{ll} \text{financially closed} & I = S \\ \text{economy} & \end{array}$$

$$\begin{array}{ll} \text{financially open} & I \geq S \\ \text{economy} & \end{array}$$

Some evidence

Data: cross-section, 130 countries

80: financially open
S0: " " closed

consider

$$(1) \quad I_t = 0.70 + 0.12 S_t \quad \bar{i} = 1 \dots 130$$

(4.3) (8.96) $\rightarrow$ t-ratios

$$(2) \quad I_t = 1.30 + 0.88 S_t \quad i = 1 \dots 56$$

(6.60) (4.92) fin closed

$$(3) \quad I_t = 1.22 - 0.12 S_t \quad \text{fin open}$$

(3.60) (-0.24) $t=80$

until now (2) is maintained. However $\rightarrow S_t$ coefficient may not be invariant to a policy regime change

Consider

$$Y_t = \beta_0 + \beta_1 X_t + U_t \quad t = \underbrace{1 \dots T_1}_{T_1}, \underbrace{T_1+1 \dots T}_{T_2}$$

$$Y_t = \beta_{01} + \beta_{11} X_t + U_t, \quad t = 1 \dots T_1$$

$$Y_t = \beta_{02} + \beta_{12} X_t + U_t, \quad t = T_1+1 \dots T$$

define

$$D_1 = \begin{cases} 1 & \text{if } t = 1 \dots T_1 \\ 0 & \text{o.w.} \end{cases}$$

$$D_2 = \begin{cases} 1 & \text{if } t = T_1+1 \dots T \\ 0 & \text{o.w.} \end{cases}$$

$$\begin{array}{c} 1 \\ 2 \\ \vdots \\ T_1 \\ T_1+1 \\ \vdots \\ T \end{array} \begin{bmatrix} Y & X & D_1 & D_2 \\ Y_1 & X_1 & 1 & 0 \\ Y_2 & X_2 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ Y_{T_1} & X_{T_1} & 1 & 0 \\ Y_{T_1+1} & X_{T_1+1} & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ Y_T & X_T & 0 & 1 \end{bmatrix}$$

note that

$$D_1 + D_2 = 1, \quad \forall t$$

Parametrisations

8st parametrisation:

$$(1) Y_t = \beta_0 D_1 + \beta_{11} D_1 X_t + \beta_{02} D_2 + \beta_{12} D_2 X_t + U_t$$

1st parametrization:

$$D_1 + D_2 = 1 \Rightarrow D_1 = 1 - D_2$$

$$Y_t = \beta_0 (1 - D_2) + \beta_{11} (1 - D_2) X_t + \beta_{02} D_2 + \beta_{12} D_2 X_t + U_t$$

$$\Rightarrow Y_t = \beta_{01} + \beta_{11} X_t + \underbrace{(\beta_{02} - \beta_{01})}_{\alpha_0} D_2 + \underbrace{(\beta_{12} - \beta_{11})}_{\alpha_1} D_2 X_t + U_t$$

$$(2) Y_t = \beta_{01} + \beta_{11} X_t + \alpha_0 D_2 + \alpha_1 D_2 X_t + U_t$$

if $\alpha_0 = 0$
 $\alpha_1 = 0$ } $\rightarrow$ the parameters remain constant over time

Base: $t = 1 \dots T_1$
 $\hookrightarrow$ regressed by the left out variable

Here: Base $\rightarrow D_1$

Dummy Variable Trap:

Adding a constant term either to (1) or (2)
 perfect multicollinearity $\leftarrow$

Since $D_1 + D_2 = 1$

$$t = \begin{matrix} 1 \\ 2 \\ \vdots \\ T_1 \\ T_1+1 \\ \vdots \\ T \end{matrix} \begin{bmatrix} Y & X & D_1 & D_2 \\ \hline Y_1 & X_1 & 1 & 0 \\ Y_2 & X_2 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ Y_{T_1} & X_{T_1} & 1 & 0 \\ Y_{T_1+1} & X_{T_1+1} & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ Y_T & X_T & 0 & 1 \end{bmatrix}$$

Chow Test

testing for a structural change:

$$H_0 : \alpha_0 = 0 \\ \alpha_1 = 0$$

$$H_A : \alpha_0 \neq 0 \\ \alpha_1 \neq 0$$

unrestricted model:

$$(2) Y_t = \beta_{01} + \beta_{11} X_t + \alpha_0 D_2 + \alpha_1 D_2 X_t + U_t$$

$$t = 1 \dots T$$

note that: (1) and (2) are observationally equivalent
they do have the same SSR's

Restricted Model

$$(3) Y_t = \beta_0 + \beta_1 X_t + U_t, \quad t = 1 \dots T$$

testing

$$F = \frac{\frac{S_3 - S_4}{S_4}}{\frac{df_u}{P}} \quad \begin{matrix} S_2 = S_1 \\ T-k-1 \end{matrix} \rightarrow 4$$

Chow test $P \rightarrow 2$

Recall

$$(A) Y_t = \beta_{01} + \beta_{11} X_t + U_t, \quad t = 1, \dots, T \Rightarrow S_A$$

$$(B) Y_t = \beta_{02} + \beta_{12} X_t + U_t, \quad t = T_1+1 \dots T \Rightarrow S_B$$

Chow shows that $S_A + S_B = S_2$

3 Categories

$$(1A) Y_t = \beta_{01} + \beta_{11} X_t + U_t \quad t = 1 \dots T_1$$

$$(2A) Y_t = \beta_{02} + \beta_{12} X_t + U_t \quad t = T_1+1 \dots T_2$$

$$(3A) Y_t = \beta_{03} + \beta_{13} X_t + U_t \quad t = T_2+1 \dots T$$

example

Y_t = consumption X_t = disposable income

T_1 = low income

T_2 = high income

T_2 = middle income

define

$$D_1 = \begin{cases} 1 & \text{if low income} \\ 0 & \text{o.w.} \end{cases}$$

$$D_2 = \begin{cases} 1 & \text{if middle income} \\ 0 & \text{o.w.} \end{cases}$$

$$D_3 = \begin{cases} 1 & \text{if high income} \\ 0 & \text{o.w.} \end{cases}$$

$$D_1 + D_2 + D_3 = 1, \forall t$$

First Parametrization

$$AA(\left[\begin{aligned} Y_t &= \beta_{01} D_1 + \beta_{11} D_1 X_t + \beta_{02} D_2 + \beta_{12} D_2 X_t + \beta_{03} D_3 + \\ &\quad \beta_{03} D_3 + \beta_{13} D_3 X_t + u_t \end{aligned} \right. \right. \\ t = 1 \dots T$$

Using $D_1 + D_2 + D_3 = 1$

$$\underbrace{D_1}_{\text{base sample}} = 1 - D_2 - D_3$$

$$Y_t = \beta_{01} + \beta_{11} X_t + \underbrace{\alpha_{01}}_{\beta_{02} - \beta_{01}} D_2 + \underbrace{\alpha_{11}}_{\beta_{12} - \beta_{11}} D_2 X_t + \underbrace{\alpha_{02}}_{\beta_{03} - \beta_{01}} D_3 + \underbrace{\alpha_{12}}_{\beta_{13} - \beta_{11}} D_3 X_t + u_t$$

testing for a structural change

$$H_0: \alpha_{01} = \alpha_{11} = \alpha_{02} = \alpha_{12} = 0$$

$$\text{Unrestricted model } AA_1 - AA_2$$

restricted model:

$$Y_t = \beta_0 + \beta_1 X_t + U_t, \quad t=1, \dots, T$$

$$F = \frac{S_R - S_U}{S_U} \cdot \frac{df_U}{p}$$

$$\begin{aligned} \text{note: } S_U &= S_{AM1} = S_{AM2} \\ &= S_{1A} + S_{2A} + S_{3A} \end{aligned}$$

F Statistic Breakdown

$$F_{cal} = \frac{R_U^2 - R_R^2}{1 - R_U^2} \cdot \frac{df_U}{q}$$

$R_U^2 \rightarrow$ coefficient of determination for the unrestricted model

$R_R^2 \rightarrow$ coefficient of determination for restricted model

$df_U \rightarrow$ degrees of freedom

$n - k_U \rightarrow$ number of parameters
number of observations

$q \rightarrow$ number of restrictions

difference in the number of parameters estimated between the unrestricted & restricted model

$$k_U - k_R$$

$$t_{cal} = \frac{\hat{\beta}_1 - \beta_1}{s.e(\hat{\beta}_1)}$$

F_{cal} with SSR's

$$\frac{SSR_e - SSR_u}{SSR_u} \cdot \frac{df_u}{9}$$