

METU, Department of Economics

ECON 201 – Problem Set 3

1- A consumer has the utility function over goods x and y ,

$$u(x, y) = x^{1/2} + y^{1/2}$$

Let the price of good x be given by P_x , let the price of good y be given by P_y , and let income be given by M .

- Derive the consumer's demand function for good y .
- What is the lagrangian coefficient λ ? What does it mean?
- Is good y normal or inferior?
- If we have $P_x = 2$, $P_y = 1$, and $M = 12$, compute the utility maximizing consumption bundle of goods x and y .

2- Emre consumes only Tea (T) and Cigarettes (C) each week with his 100TL income. Tea costs 1TL, while cigarettes cost only 0.50TL. Emre has Cobb-Douglas preferences given by:

$U(T, C) = T(C - c)$ where c is the quantity of cigarettes that his neighbor Ekin consumes. Assume throughout this question that Emre always consumes more cigarettes than Ekin.

- How does Ekin's cigarette consumption influence Emre's utility function? Specifically, compute and determine the sign of $\partial U / \partial c$. Intuitively, what does this tell you about Emre's happiness and Ekin's consumption of cigarettes?
- Holding c fixed, compute Emre's optimal consumption bundle of Tea and Cigarettes as functions of Ekin's consumption of cigarettes, c .
- How does Ekin's consumption of cigarettes affect Emre's optimal bundle? Specifically calculate and determine the sign of $\partial T^* / \partial c$ and $\partial C^* / \partial c$ where (T^*, C^*) is Emre's optimal consumption bundle.

3- If preferences are concave what does this imply in terms of the MRS? In such a case, will the consumer ever consume both of the goods together?

4- Suppose that there are two goods (a and b) in an economy and a consumer with the following utility function:

$$u(a, b) = 10a - a^2 + b$$

Assume that price of a is P_a and price of P_b is 1.

- Sketch two indifference curves of this consumer.
- How many a would the household demand at $p_a = 0$?

c) At what price would the household be indifferent between $a - 1$ and a ?

5- A consumer has a utility function:

$$u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$$

- a) Derive the Marshallian demands of both goods.
- b) Derive the income expansion path (i.e., income offer curve).
- c) Derive the Engel curve for both goods.

6- A consumer derives utility from consuming x_1 and x_2 . His utility function is

$$u(x_1, x_2) = 8\sqrt{x_1 x_2}$$

Suppose he wants to have exactly $u = \bar{u}$ given P_1 and P_2 .

- a) Formulate the constrained expenditure minimization problem.
 - b) Determine the optimum quantities of x_1 and x_2 in order to achieve $\bar{u}$ given prices P_1 and P_2 (i.e. find compensated/Hicksian demand function).
 - c) What is his minimum expenditure while receiving utility level of $\bar{u}$?
- 7- Suppose that the consumer now faces a choice between drinking tea and coffee, from which she obtains the utility $u(c, t) = \ln(3t + c)$.
- a) What are the Marshallian demand functions of tea and coffee, given their prices as P_t and P_c , respectively.
 - b) What is the income expansion path?
 - c) Derive the Engel curves of both goods.
- 8- Use the Slutsky equation to show that a Giffen good must be an inferior good, but an inferior good need not be a Giffen good. Show the latter case graphically.
- 9- Suppose Rafael spends his monthly stipend only on ice cream (M) and chocolate (C) and his utility function for these goods is $U(M, C) = MC$. Rafael has a monthly income of 60€ and the prices are $P_M = 5€$ & $P_C = 6€$. What will be the income and substitution effects of an increase of 1.5 € in the price of chocolate?