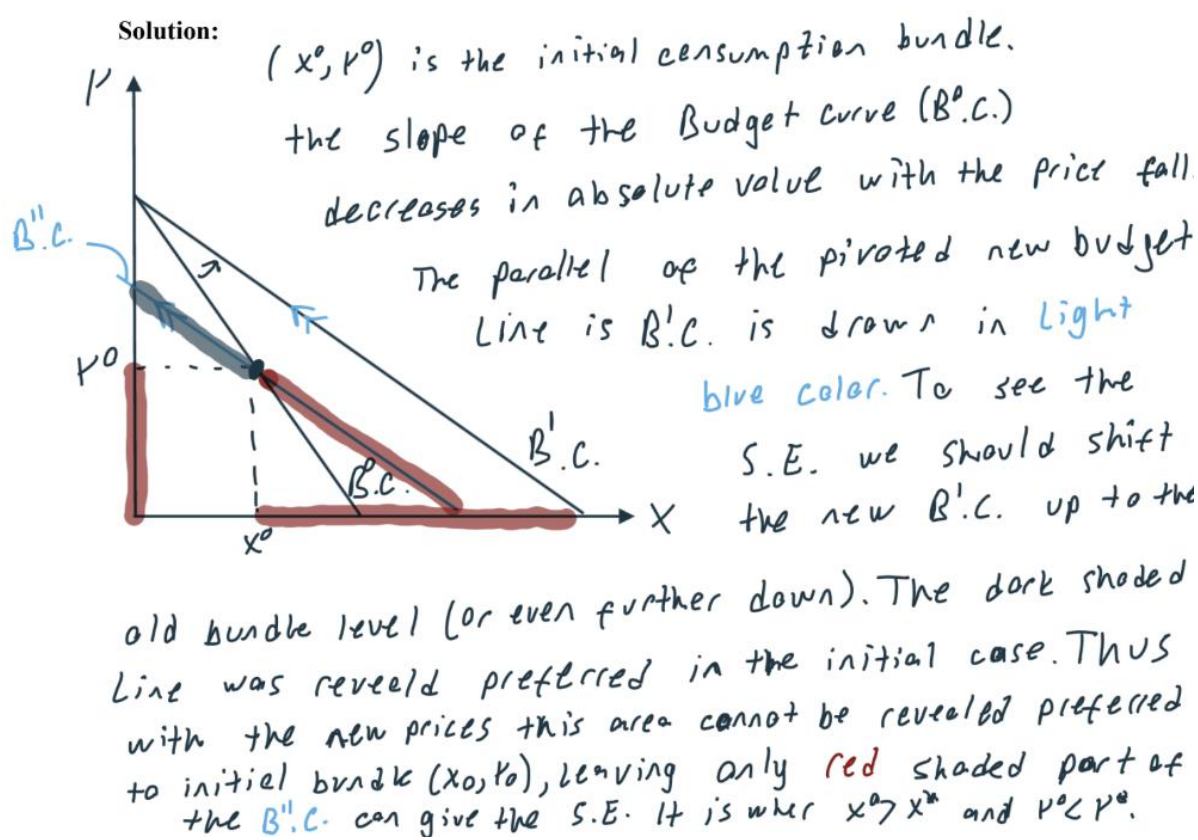


METU, Department of Economics

ECON 201 – Problem Set 5

- 1- Use a diagram to prove that in case there are two goods, the substitution effect of a decrease in the price of good x SHOULD increase the demand for good x for an individual who satisfies WARP. (Hint: First, draw the budget line which is pivoted due to the price change. Second, shift the budget line so that the initial bundle is still affordable. Lastly, show the new possible bundles which does not violate WARP and comment.)



- 2- Carlos has at one time or another lived in Argentina, Bolivia, and Colombia. He buys only two goods, x and y . In Argentina the prices were (\$9, \$3) and he consumed the bundle (6, 7). In Bolivia he consumed (9, 2). In Colombia he consumed the bundle (6, 5) at the prices (\$3, \$3). Comment on the following claims (True/False and explain why)
- The Argentine bundle is directly revealed preferred to the Bolivian bundle.
 - The Argentine bundle is indirectly revealed preferred to the Bolivian bundle.
 - The Colombian bundle is directly revealed preferred to the Argentine bundle.

Solution:

	<u>A</u>	<u>B</u>	<u>C</u>
Bundle:	(6,7)	(9,2)	(6,5)
Prices:	(9,3)	?	(3,3)
Total Exp.:	<u>75 \$</u>		<u>33 \$</u>

At Argentina, Colombia's bundle (6,5) is affordable.
 At Colombia, Bolivia's bundle (9,2) is affordable.
 → A is revealed preferred to C (directly)
 → C is revealed preferred to B (directly)
 ⇒ transitivity → A is revealed preferred to B (indirectly)

- 3- At prices $(p_1, p_2) = (\$4, \$2)$, Ivan buys the bundle $(x_1, x_2) = (8, 20)$.
 At prices $(p_1', p_2') = (\$2, \$4)$, he buys the bundle $(x_1', x_2') = (10, 13)$.
 At prices (p_1'', p_2'') , he buys the bundle $(x_1'', x_2'') = (14, 11)$.
 Assuming that his preferences satisfy the strong axiom of revealed preferences, find a relation between P_1'' and P_2'' . (i.e. something like $aP_1'' < bP_2''$). Show your work.

Solution:

	<u>①</u>	<u>②</u>	<u>③</u>
Bundle:	(8,20)	(10,13)	(14,11)
Prices:	(4,2)	(2,4)	(P_1'', P_2'')
Total exp.:	72	72	—

With the prices (4,2): (8,20) and (10,13) are affordable.
 ⇒ (8,20) is directly revealed preferred to (10,13)

with the prices (2,4): (10, 13) and (14, 11) are affordable.

$\Rightarrow$ (10, 13) is directly revealed preferred to (14, 11).

$\Rightarrow$ (8, 20) is indirectly " " to (14, 11)

WARP: If some bundle x is directly revealed preferred to bundle y . Then, it cannot happen that y is directly revealed preferred to x .

SARP: If some bundle (either directly or indirectly) rev. preferred to bundle y . Then, it cannot happen that y is (directly or indirectly) rev. pref. to x .

WARP: $10p_1'' + 13p_2'' > 14p_1'' + 11p_2'' \Rightarrow \cancel{10}p_1'' < \cancel{2}p_2''$
requires $\Rightarrow \underline{\underline{2p_1'' < p_2''}}$

SARP: $8p_1'' + 20p_2'' > 14p_1'' + 11p_2'' \Rightarrow \cancel{8}p_1'' < \cancel{3}p_2''$
requires

- 4- If the production function is $f(x, y) = x + \min\{x, y\}$, then there are constant returns to scale. T/F?

True; $f(tx, ty) = tx + \min\{tx, ty\} = tx + t \min\{x, y\}$
 $\Rightarrow f(tx, ty) = t f(x, y)$

- 5- A firm has two variable factors and a production function $f(x_1, x_2) = (2x_1 + 4x_2)^{1/2}$. Find the technical rate of substitution between x_1 and x_2 .

$$MRTS = - \frac{MP_1(x_1, x_2)}{MP_2(x_1, x_2)} = - \frac{\frac{1}{2} \cdot 2 (2x_1 + 4x_2)^{-1/2}}{\frac{1}{2} \cdot 4 (2x_1 + 4x_2)^{-1/2}} = - \frac{1}{2}$$

- 6- A firm has the production function $f(x_1, x_2) = (x_1^b + x_2^b)^c$ where $b > 0$ and $c > 0$. Find a relation between b and c (such as an inequality) for which this firm will have increasing returns to scale.

$$f(x_1, x_2) = (x_1^b + x_2^b)^c, \quad b > 0, \quad c > 0$$

$$\begin{aligned} f(kx_1, kx_2) &= ((kx_1)^b + (kx_2)^b)^c = (k^b(x_1^b + x_2^b))^c \\ &= k^{bc} (x_1^b + x_2^b)^c \\ &= k^{bc} f(x_1, x_2) \end{aligned}$$

$\Rightarrow$ if $bc > 1$ then firm has an increasing returns to scale

- 7- Suppose a firm's production function is given as
 $Q(K, L) = 150K^2L^2 - K^3L^3$

Assume $K = 5$ is fixed.

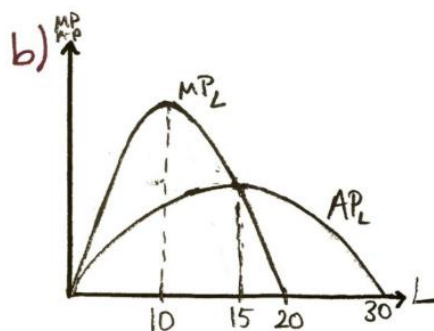
- Derive the firm's total product of labor, marginal product of labor and average product of labor.
- Plot MPL and APL on the same graph.
- Find the amount of labor at which MPL is maximized.
- Find the amount of labor necessary for this production process for $MPL = APL$.
- What is the maximum number of output the firm can produce in the short run?

$$Q(K, L) = 150K^2L^2 - K^3L^3 \quad \bar{K} = 5$$

$$a) \text{ TPL}(L) = 150 \cdot 25 \cdot L^2 - 125L^3 = 3750L^2 - 125L^3$$

$$\text{MPL}(L) = \frac{\partial \text{TPL}(L)}{\partial L} = 7500L - 375L^2$$

$$\text{APL}(L) = \frac{\text{TPL}}{L} = 3750L - 125L^2$$



$$c) \frac{\partial \text{MPL}}{\partial L} = 0 \Rightarrow 7500L - 7500L = 0 \Rightarrow L = 10$$

$$d) \text{MPL} = \text{APL} \Rightarrow 3750L = 250L^2 \Rightarrow L = 15$$

$$e) \frac{\partial \text{TPL}}{\partial L} = 0 \Rightarrow 7500L - 375L^2 = 0 \Rightarrow 375L(20 - L) = 0$$

$$\Rightarrow L = 0 \text{ or } L = 20$$

$$\begin{aligned} L = 20 &\Rightarrow Q = 500,000 \\ L = 0 &\Rightarrow Q = 0 \end{aligned} \Rightarrow L^* = 20$$

- 8- Suppose a firm's production function is given as $f(K, L) = \sqrt{8L}$ with input prices r, w and p denotes the output price and assume that K is fixed $K = 1$.
- Write down the profit maximization problem.
 - Derive the optimal production plan which maximizes the firm's profit.
 - Find the firm's maximized profit level.
 - How does the firm's optimal output level change if wage increases or output price p decrease?

$$a) \text{ Max } \pi = p \cdot f(K, L) - r\bar{K} - wL$$

$$\text{Max } p\sqrt{8L} - r - wL$$

$$b) \text{ From F.O.C; } \frac{4P}{\sqrt{8L}} - w = 0 \Rightarrow L^* = \frac{2P^2}{w^2}$$

$$\text{Thus } f(\bar{K}, L^*) = \sqrt{8L^*} = \frac{4P}{w}$$

$$c) \pi = p \cdot \frac{4P}{w} - r \cdot 1 - w \frac{2P^2}{w^2} = \frac{2P^2 - wr}{w}$$

$$d) \frac{\partial L^*}{\partial w} < 0$$

$$\frac{\partial Q}{\partial w} < 0$$

If wage increases, L^* decreases and optimal output level decreases.

$$\frac{\partial L^*}{\partial P} > 0$$

If price decreases, L^* and optimal output levels decrease.

$$\frac{\partial Q}{\partial P} > 0$$