

- suggested Solutions -

① $c_1(y_1) = y_1^2$
 $c_2(y_2) = 12y_2^2$
 $P = 100 - y$

(a) Problem of firm 1: $\max \pi_1 = (100 - y_1 - y_2)y_1 - y_1^2$

$$\partial \pi_1 / \partial y_1 = 0 \Rightarrow 4y_1 + y_2 = 100$$

Best-response function of firm 1: $y_1 = \frac{100 - y_2}{4}$

Problem of firm 2: $\max \pi_2 = (100 - y_1 - y_2)y_2 - 12y_2^2$

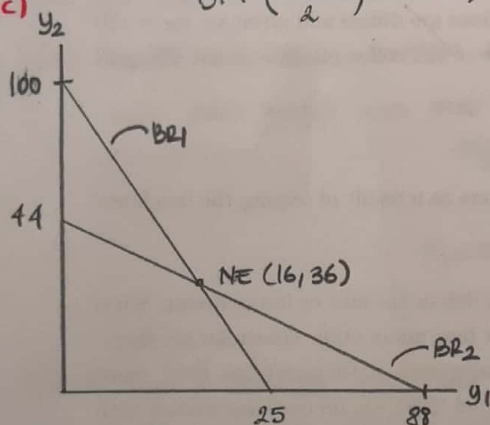
$$\partial \pi_2 / \partial y_2 = 0 \Rightarrow y_1 + 2y_2 = 88$$

Best-response function of firm 2: $y_2 = \frac{88 - y_1}{2}$

(b) Cournot - Nash Eqm.

solve the system of best-response eqns.

(c) $4y_1 + \frac{(88 - y_1)}{2} = 100 \Rightarrow \begin{cases} y_1^* = 16 \\ y_2^* = 36 \end{cases} \Rightarrow y^* = 52$



② $C = \$50$
 $Q = 100 - P$

Notice that if $P_1 < P_2$, firm 2 will be pushed out of market ($Q_1 = Q$ and $Q_2 = 0$)

if $P_1 > P_2$ again, no one will buy from firm 1 ($Q_1 = 0$ and $Q_2 = Q$)

So, $P_1 = P_2$ is the best response func. for both; $Q_1 = Q_2 = Q$

So, both firms will cut prices not to lose the mkt. until $P = MC$.

$$Q = 100 - 50 = 50 \quad Q_1 = Q_2 = 25$$

③ n identical firms: $P = \alpha - \beta \cdot Q$

Firm i 's problem: $\max \pi_i = \left(\alpha - \beta \cdot \sum_{j=1}^n q_j \right) \cdot q_i - c \cdot q_i$

$$\partial \pi_i / \partial q_i = 0 \Rightarrow \alpha - \beta \cdot \sum_{j \neq i}^n q_j - 2\beta q_i - c = 0$$

$$BR_i = q_i^* = \frac{\alpha - \beta \cdot \sum_{j \neq i}^n q_j - c}{2\beta}$$

①

Since firms are identical, best responses will be identical. So, plug in q_i for all n firms to find the Nash-eqm.

$$\sum_{i \neq j}^n q_j = (n-1) \cdot q_i$$

$$q_i^* = \frac{\alpha - \beta(n-1)q_i^* - c}{2\beta} \Rightarrow q_i^* = \frac{\alpha - c}{\beta(n+1)}$$

$$\pi_i^* = \left(\alpha - \beta n \frac{(\alpha - c)}{\beta(n+1)} \right) \frac{\alpha - c}{\beta(n+1)} - c \cdot \frac{\alpha - c}{\beta(n+1)}$$

$$\pi^* = \left(\frac{\alpha - c}{n+1} \right)^2 \cdot \frac{1}{\beta}$$

$$\cdot \text{ If } n \rightarrow \infty, \lim_{n \rightarrow \infty} \pi^* = \lim_{n \rightarrow \infty} \left(\frac{\alpha - c}{n+1} \right)^2 \cdot \frac{1}{\beta} = 0$$

As # of firms $\uparrow$, profit converges to zero.

④ $P = 400 - 2Q \quad \wedge \quad c = \40

(a) Cartel $\Rightarrow$ Firms behave as if they're monopoly together.

$$\text{max joint profit} \Rightarrow \text{max } \pi = (400 - 2Q)Q - 40Q$$

$$\frac{\partial \pi}{\partial Q} = 0 \Rightarrow Q = 90 \quad q_1 = q_2 = 45$$

$$P = \$220$$

$$\pi_1 = \pi_2 = \$220 \cdot 45 - \$40 \cdot 45 = \$8100$$

(b) If firm 1 assumes that firm 2 will maintain the cartel level of production, firm 1 faces the following demand curve:

$$P = 400 - 2(q_1 + 45)$$

$$\text{max } \pi_1 = (400 - 2(q_1 + 45))q_1 - 40q_1$$

$$\frac{\partial \pi_1}{\partial q_1} = 0 \Rightarrow q_1 = 67.5$$

$$\pi_1 = 9112.50 \quad \text{and} \quad \pi_2 = 6075$$

Notice that by cheating, firm 1 will make more profit.

(c) Now find what happens if they both cheat:

$$\text{max } \pi = (400 - 2q_1 - 2q_2)q_1 - 40q_1$$

$$\frac{\partial \pi}{\partial q_1} = 0 \quad q_1 = \frac{180 - q_2}{2} \quad q_1^* = q_2^* = 60$$

$$\pi_1 = \pi_2 = 7200$$

(cheating is the dominant strategy for both firms)

		F2	
		Coop.	Cheat
F1	Coop.	8100, 8100	6075, 9112
	Cheat	9112, 6075	7200, 7200

⑤ Player 1's best response correspondence is the solution to the following problem:

$$\max_{e_1 \geq 0} \quad \delta_1 e_1 + \beta e_1 e_2 - \frac{1}{2} e_1^2$$

$$FOC(e_1): \quad \delta_1 + \beta \cdot e_2 - e_1 = 0$$

$$BR_1 = e_1 = \delta_1 + \beta \cdot e_2$$

similarly, the best response correspondence of pl 2: $BR_2(e_1) = \delta_2 + \beta \cdot e_1$

Therefore, any NE must satisfy the following two eqns:

$$\left. \begin{aligned} e_1 &= \delta_1 + \beta \cdot e_2 \\ e_2 &= \delta_2 + \beta \cdot e_1 \end{aligned} \right\} \quad e_1 = \frac{\delta_1 + \beta \delta_2}{1 - \beta^2} \quad \text{and} \quad e_2 = \frac{\delta_2 + \beta \delta_1}{1 - \beta^2}$$

⑥ $\max \pi_1 = (24 - 5p_1 + 2p_2)p_1$

$$\frac{\partial \pi_1}{\partial p_1} = 0 \quad p_1 = \frac{1}{10} (24 + 2p_2) = BR_1$$

$$\max \pi_2 = (24 - 5p_2 + 2p_1)p_2$$

$$\frac{\partial \pi_2}{\partial p_2} = 0 \quad p_2 = \frac{1}{10} (24 + 2p_1)$$

• Solve together to find NE: $p_1^* = p_2^* = 3$ and $\pi_1^* = \pi_2^* = 45$

• Collusion: $\max \text{ joint } \pi = (24 - 5p_1 + 2p_2)p_1 + (24 - 5p_2 + 2p_1)p_2$

$$\left. \begin{aligned} \frac{\partial \pi}{\partial p_1} = 0 &\Rightarrow p_1 = \frac{24 + 4p_2}{10} \\ \frac{\partial \pi}{\partial p_2} = 0 &\Rightarrow p_2 = \frac{24 + 4p_1}{10} \end{aligned} \right\} \quad \begin{aligned} p_1 &= p_2 = 4 \\ \pi_1 &= \pi_2 = 48 \end{aligned}$$

• Yes, there is an incentive to deviate. If firm 2 sticks to price 4, then firm 1 would set price such that:

$$\text{recall } BR_1 = p_1 = \frac{1}{10} (24 + 2 \cdot 4)$$

$$p_1^d = 3.2$$

$$\pi_1^d = 51.2$$

since $\pi^d > \pi$, firms tend to cheat.