

2025-2026

Problem Set I - Answers

Q1)

a) Two cards are drawn from a standard 52-card playing deck. What is the probability that the draw will yield an ace and a face card?

SOLUTION

There are $\binom{52}{2} = 1326$ ways to draw two cards from the deck. The probability is $4 \cdot 12 / 1326 = 0.0362$.

b) Five cards are dealt from a standard 52-card deck. What is the probability that we draw 3 aces and 2 kings?

SOLUTION

There are $\binom{52}{5} = 2598960$ ways to draw five cards from the deck. There are $\binom{4}{3} \cdot \binom{4}{2} = 24$ ways to draw three Aces and two kings. The probability is $24 / 2598960$.

c) Five cards are dealt from a standard 52-card deck. What is the probability that we draw 1 ace, 1 two, 1 three, 1 four, and 1 five?

SOLUTION

$\binom{4}{1} \cdot \binom{4}{1} \cdot \binom{4}{1} \cdot \binom{4}{1} \cdot \binom{4}{1} = 4^5 = 1024$. The probability is $1024 / 2598960$.

Q2) Cards are dealt, one at a time, from a standard 52-card deck. If the first 2 cards are both spades, what is the probability that the next three cards are also spades?

SOLUTION

There are 13 spades in a deck. Given the 2 cards are spades, there are 11 spades left in the deck. The probability is $\frac{\binom{11}{3}}{\binom{50}{3}} = 0.0084$.

Q3) Suppose economists are analyzing current economic conditions in two U.S. regions: the West Coast (California) and the East Coast (Florida). Let A represent the event that the unemployment rate in California decreases as projected this quarter, and let B represent the event that consumer confidence in Florida improves as expected this quarter. Assume that A and B are independent events and that $P(A) > P(B)$. Additionally, it is known that

$$P(A \cup B) = 0.626 \text{ and } P(A \cap B) = 0.144$$

Determine the probabilities $P(A)$ and $P(B)$.

SOLUTION

Apply the addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow .626 = P(A) + P(B) - .144$. Apply independence: $P(A \cap B) = P(A)P(B) = .144$.

So, $P(A) + P(B) = .770$ and $P(A)P(B) = .144$.

Let $x = P(A)$ and $y = P(B)$. Using the first equation, $y = .77 - x$, and substituting this into the second equation yields $x(.77 - x) = .144$ or $x^2 - .77x + .144 = 0$. Use the quadratic formula to solve:

$$x = \frac{.77 \pm \sqrt{(-.77)^2 - (4)(1)(.144)}}{2(1)} = \frac{.77 \pm .13}{2} = .32 \text{ or } .45. \text{ Since } x = P(A) \text{ is assumed to be the larger}$$

probability, $x = P(A) = .45$ and $y = P(B) = .32$.

Q4) Same product is produced by machines A, B and C. %5 of the products produced by Machine A, %4 of the product produced by Machine B and %3 of the products produced by Machine C is broken. In a single day %25 of the products are produced by Machine A, %30 of the products are produced by Machine B and %45 of the products are produced by Machine C. A product produced in a single day is randomly selected and known to be broken then what is the probability that that product is produced by Machine C.

SOLUTION

Apply Bayes' rule:

$H = \{ \text{Selected product is broken} \}$

$A = \{ \text{Product is produced by Machine A} \}$

$B = \{ \text{Product is produced by Machine B} \}$

$C = \{ \text{Product is produced by Machine C} \}$

$P(A) = 0.25, P(H|A) = 0.05, P(B) = 0.30, P(H|B) = 0.04, P(C) = 0.45, P(H|C) = 0.03$

$$P(H) = P(A)P(H|A) + P(B)P(H|B) + P(C)P(H|C) = \frac{1}{4} \frac{1}{20} + \frac{3}{10} \frac{1}{25} + \frac{9}{20} \frac{3}{100} = \frac{19}{500}$$

$$P(C|H) = \frac{P(C \cap H)}{P(H)} = \frac{P(C)P(H|C)}{P(H)} = \frac{(9/20)(3/100)}{(19/500)} = \frac{27}{76}$$

Q5) Medical case histories indicate that different illnesses may produce identical symptoms. Suppose that a particular set of symptoms, denoted H , occurs only when any one of three illnesses, I_1, I_2 , or I_3 , occurs. Assume that the simultaneous occurrence of more than one of these illnesses is impossible and that

$$P(I_1) = .01, P(I_2) = .005, P(I_3) = .02.$$

The probabilities of developing the set of symptoms H , given each of these illnesses, are known to be

$$P(H | I_1) = .90, P(H | I_2) = .95, P(H | I_3) = .75$$

Assuming that an ill person exhibits the symptoms, H , what is the probability that the person has illness I_1 ?

SOLUTION

Using Bayes' rule,

$$P(I_1 | H) = \frac{P(H | I_1)P(I_1)}{P(H | I_1)P(I_1) + P(H | I_2)P(I_2) + P(H | I_3)P(I_3)} = 0.313.$$

Q6) The undergraduate statistics club at a certain university has 24 members.

- All 24 members of the club are eligible to attend a conference next week, but they can only afford to send 4 people. In how many possible ways could 4 attendees be selected?
- All club members are eligible for any of the four positions of president, VP, secretary, or treasurer. In how many possible ways can these positions be occupied?
- Suppose it's agreed that two people will be cochairs, one person secretary, and one person treasurer. How many ways are there to fill these positions now?

SOLUTION

a. $\binom{24}{4} = 10,626.$

b. Order matters here (being selected as VP isn't the same as Treasurer): ${}_{24}P_4 = (24)(23)(22)(21) = 255,024.$

c. There are $\binom{24}{2} = 276$ ways to choose co-chairs, then 22 choices for secretary and 21 for treasurer. Apply the Fundamental Counting Principle: $(276)(22)(21) = 127,512$ (exactly half of b).

Q7) One needs to distribute 20 labors to four different construction jobs. The first job required 6 labors, the second, third and fourth utilized 4, 5 and 5 labors, respectively. Find the probability of the observed events assuming that the labors are randomly assigned to jobs.

a) All 4 members of a particular ethnic group are placed on job 1.

The number of ways to assign 20 labors to 4 jobs is $\binom{20}{6\ 4\ 5\ 5} = \frac{20!}{6!4!5!5!} = N$

All 4 members are assigned to job 1. We need to place the remaining 16. $\binom{16}{2\ 4\ 5\ 5} = \frac{16!}{2!4!5!5!} = n_a$

The probability is $\frac{n_a}{N} = 0.0031$.

b) An ethnic group member is assigned to each type of job.

$$n_b = \binom{4}{1\ 1\ 1\ 1} \binom{16}{5\ 3\ 4\ 4}$$

The probability is $\frac{n_b}{N} = 0.1238$.

Q8) A local fraternity is conducting a raffle where 50 tickets are to be sold- one per customer. There are three prizes to be rewarded. If the four organizers of the raffle each buy one ticket, what is the probability that the four organizers win

- All the prizes?
- Exactly two of the prizes?
- Exactly one of the prizes?
- None of the prizes?

SOLUTION

There are $\binom{50}{3} = 19,600$ ways to choose the 3 winners. Each of these is equally likely.

a. There are $\binom{4}{3} = 4$ ways for the organizers to win all of the prizes. The probability is $4/19600$.

b. There are $\binom{4}{2} \binom{46}{1} = 276$ ways the organizers can win two prizes and one of the other 46 people to win the third prize. So, the probability is $276/19600$.

c. $\binom{4}{1} \binom{46}{2} = 4140$. The probability is $4140/19600$.

d. $\binom{46}{3} = 15,180$. The probability is $15180/19600$.

Q9) We know the following about a colormetric method used to test lake water for nitrates. If water specimens contain nitrates, a solution dropped into the water will cause the specimen to turn red 95% of the time. When used on water specimens without nitrates, the solution causes the water to turn red 10% of the time (because chemicals other than nitrates are sometimes present and they also react to the agent). Past experience in a lab indicates that nitrates are contained in 30% of the water specimens that are sent to the lab for testing. If a water specimen is randomly selected

- a) from among those sent to the lab, what is the probability that it will turn red when tested?
- b) and turns red when tested, what is the probability that it actually contains nitrates?

SOLUTION

Let R denote the event that the specimen turns red and N denote the event that the specimen contains nitrates.

a. $P(R) = P(R | N)P(N) + P(R | \bar{N})P(\bar{N}) = .95(.3) + .1(.7) = 0.355.$

b. Using Bayes' rule, $P(N|R) = .95(.3)/.355 = 0.803.$

Q10) A monkey is to demonstrate that she recognize colors by tossing one red, one black and one white ball into boxes of the same respective colors, one ball to a box. If the monkey has not learned the colors and merely tosses one ball into each box at random, find the probability of the following results:

- There are no colors matches.
- There is exactly one color match.

SOLUTION

Solution This problem can be solved by listing sample points because only three balls are involved, but a more general method will be illustrated. Define the following events:

- A_1 : A color match occurs in the red box.
- A_2 : A color match occurs in the black box.
- A_3 : A color match occurs in the white box.

There are $3! = 6$ equally likely ways of randomly tossing the balls into the boxes with one ball in each box. Also, there are only $2! = 2$ ways of tossing the balls into the boxes if one particular box is required to have a color match. Hence,

$$P(A_1) = P(A_2) = P(A_3) = 2/6 = 1/3.$$

Similarly, it follows that

$$P(A_1 \cap A_2) = P(A_1 \cap A_3) = P(A_2 \cap A_3) = P(A_1 \cap A_2 \cap A_3) = 1/6.$$

We can now answer parts (a) and (b) by using the event-composition method.

a Notice that

$$\begin{aligned} P(\text{no color matches}) &= 1 - P(\text{at least one color match}) \\ &= 1 - P(A_1 \cup A_2 \cup A_3) \\ &= 1 - [P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) \\ &\quad - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)] \\ &= 1 - [3(1/3) - 3(1/6) + (1/6)] = 2/6 = 1/3. \end{aligned}$$

b We leave it to you to show that

$$\begin{aligned} P(\text{exactly one match}) &= P(A_1) + P(A_2) + P(A_3) \\ &\quad - 2[P(A_1 \cap A_2) + P(A_1 \cap A_3) + P(A_2 \cap A_3)] \\ &\quad + 3[P(A_1 \cap A_2 \cap A_3)] \\ &= (3)(1/3) - (2)(3)(1/6) + (3)(1/6) = 1/2. \quad \blacksquare \end{aligned}$$