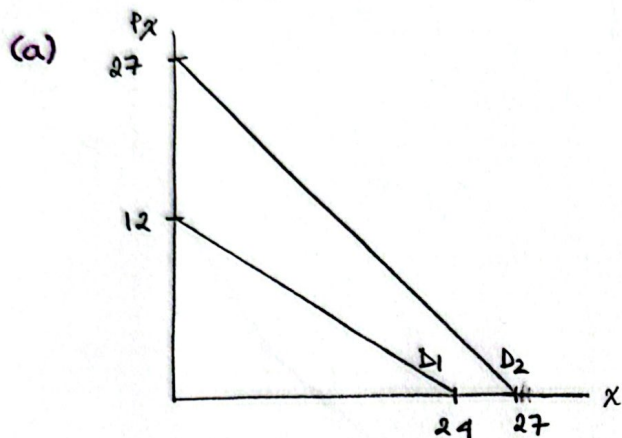


EWON 201.01 PS #1 SUGGESTED SOLUTIONS

① $X_1 = 10 - 2P_X + 0.01 I_1 + 0.4 P_Y$

$X_2 = 5 - P_X + 0.02 I_2 + 0.2 P_Y$



• Draw the demand curves for
 $I_1 = 1000$, $I_2 = 1000$, $P_Y = 10$

$X_1 = 10 - 2P_X + 0.01 \times 1000 + 0.4 \times 10$

$X_1 = 10 - 2P_X + 10 + 4 = 24 - 2P_X$

• If $P_X = 0 \Rightarrow X_1 = 24$

$X_1 = 0 \Rightarrow P_X = 12$

$X_2 = 5 - P_X + 0.02 \times 1000 + 0.2 \times 10$

$X_2 = 5 - P_X + 20 + 2 = 27 - P_X$

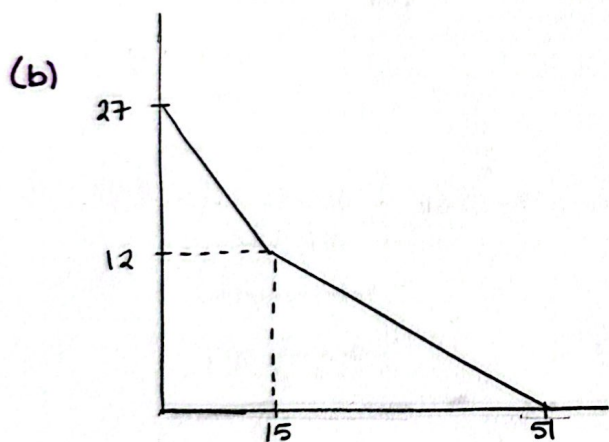
• If $P_X = 0 \Rightarrow X_2 = 27$

$X_2 = 0 \Rightarrow P_X = 27$

— • —

• Mkt demand

$X = X_1 + X_2 = 51 - 3P_X$ (valid for $P_X \leq 12$,
 above 12 only X_2 buys)



• $X(P_X) = \begin{cases} 51 - 3P_X & , P_X \leq 12 \\ 27 - P_X & , 12 < P_X \leq 27 \\ 0 & , P_X > 27 \end{cases}$

②

(a)

$$Q_A = 5 - P \quad (\text{for } P \leq 5)$$

$$Q_B = 2 - \frac{1}{5}P \quad (\text{for } P \leq 10)$$

$$Q_C = 20 - P \quad (\text{for } P \leq 20)$$

$$Q_D = 5 - \frac{1}{2}P \quad (P \leq 10)$$

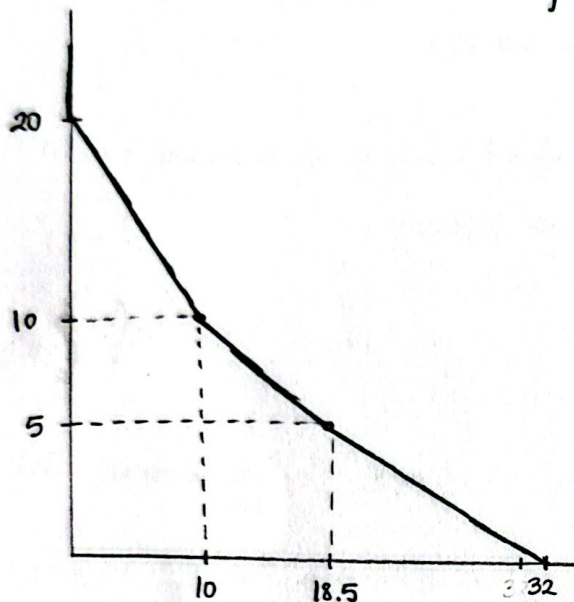
Aggregate by price ranges:

$$10 < P \leq 20 : Q = Q_C = 20 - P$$

$$5 < P \leq 10 : Q = Q_B + Q_C + Q_D = \left(2 - \frac{1}{5}P\right) + (20 - P) + \left(5 - \frac{1}{2}P\right) \\ = 27 - \frac{17}{10}P$$

$$P \leq 5 : Q = Q_A + Q_B + Q_C + Q_D = (5 - P) + \left(2 - \frac{1}{5}P\right) + (20 - P) + \left(5 - \frac{1}{2}P\right) \\ = 32 - \frac{27}{10}P$$

(of course, when $P > 20 \Rightarrow Q = 0$)



$$(b) \cdot \left. \begin{array}{l} 10 < P \leq 20 \\ 0 \leq Q \leq 10 \end{array} \right\} Q = 20 - P \Rightarrow \epsilon_{D,P} = \frac{dQ}{dP} \cdot \frac{P}{Q} \quad \text{and} \quad \frac{dQ}{dP} = -1$$

$$1 < \frac{P}{Q} < \infty$$

$$1 < |\epsilon_{D,P}| < \infty$$

→ the demand is most elastic

$$\cdot \left. \begin{array}{l} 5 < P \leq 10 \\ 10 < Q \leq 18.5 \end{array} \right\} Q = 27 - \frac{17}{10}P \Rightarrow \frac{dQ}{dP} = -\frac{17}{10}$$

$$\frac{5}{18.5} < \frac{P}{Q} \leq 1$$

$$\frac{17}{37} < |\epsilon_{D,P}| < 1.7$$

$$\cdot \left. \begin{array}{l} 0 \leq P \leq 5 \\ 18.5 \leq Q \leq 32 \end{array} \right\} Q = 32 - \frac{27}{10}P \Rightarrow \frac{dQ}{dP} = -\frac{27}{10}$$

$$0 \leq \frac{P}{Q} \leq \frac{5}{18.5}$$

$$\frac{dQ}{dP} = -\frac{27}{10}$$

$$0 \leq |\epsilon_{D,P}| \leq \frac{27}{37} \rightarrow \text{the demand is most inelastic.}$$

②

(c) • $0 \leq P \leq 5 \Rightarrow Q = 32 - 2.7P$

$$32 - 2.7P = -6 + \frac{3}{2}P$$

$$P \approx 7.61 \Rightarrow \text{Price must be } 0 \leq P \leq 5 \Rightarrow \text{CONTRADICTION } \nabla$$

• $5 < P \leq 10 \Rightarrow 27 - 1.7P = -6 + \frac{3}{2}P$

$$P \approx 10.3 \Rightarrow \text{CONTRADICTION } \nabla$$

• $10 < P \leq 20 \Rightarrow 20 - P = -6 + \frac{3}{2}P$

$$P \approx 10.4 \Rightarrow \text{So, only in this segment intersection occurs \& only Chris buy widgets.}$$

(d) slope = $\frac{\Delta P}{\Delta Q}$

• As $P \downarrow \Delta \uparrow$ according to the law of demand (for normal goods)
and also as P decreases we sum individual demands, so the coefficient of P increases
(which is the inverse of the slope) $\Rightarrow$ so demand curve gets flatter.

③ $Q = \beta \cdot P^\alpha$

(a) $Q_d \geq 0$ which means that $\beta \cdot P^\alpha \geq 0$; so $P^\alpha \geq 0$ and $\beta \geq 0$

$$\partial Q_d / \partial P < 0 \rightarrow Q \text{ is negatively related with } P$$

$$\frac{\partial Q}{\partial P} = \alpha \cdot \beta \cdot P^{\alpha-1} < 0 \quad . \quad \text{Given } \beta \geq 0 \text{ and } P^\alpha \geq 0, \text{ then } \alpha < 0.$$

(b) $\epsilon_{D,P} = \frac{\partial Q_d}{\partial P} \cdot \frac{P}{Q_d} = \alpha \cdot \beta \cdot P^{\alpha-1} \cdot \frac{P}{\beta \cdot P^\alpha} = \alpha \Rightarrow \epsilon_{D,P} \text{ does not depend on } P \text{ and } Q.$

(c) $Q_d \geq 0 \Rightarrow \gamma \cdot P^\alpha \cdot I^\beta \geq 0$ and $\gamma \geq 0$ [Assuming $P > 0$ and $I > 0$]

$$\partial Q_d / \partial P < 0 \Rightarrow \gamma \cdot \alpha \cdot P^{\alpha-1} \cdot I^\beta < 0 \rightarrow \text{we know } \gamma \geq 0; \text{ so } \alpha < 0$$

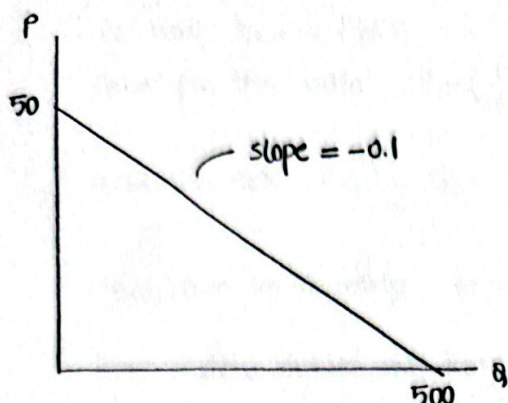
$$\partial Q_d / \partial I > 0 \Rightarrow \gamma \cdot \beta \cdot P^\alpha \cdot I^{\beta-1} > 0 \Rightarrow \text{we know } \gamma \geq 0; \text{ then } \beta > 0.$$

(d) $\epsilon_{D,P} = \frac{\partial Q_d}{\partial P} \cdot \frac{P}{Q_d} = \gamma \cdot \alpha \cdot P^{\alpha-1} \cdot I^\beta \cdot \frac{P}{\gamma \cdot P^\alpha \cdot I^\beta} = \alpha$

$$\epsilon_{D,I} = \frac{\partial Q_d}{\partial I} \cdot \frac{I}{Q_d} = \gamma \cdot \beta \cdot P^\alpha \cdot I^{\beta-1} \cdot \frac{I}{\gamma \cdot P^\alpha \cdot I^\beta} = \beta$$

③

④ Let $Q = 500 - 10P$



$$E_{D,P} = \frac{dQ}{dP} \cdot \frac{P}{Q}$$

↓
inverse
of slope
= -10

; then $E_{D,P} = -10 \cdot \frac{P}{Q}$

• at $P=10$ $Q=400 \Rightarrow E = -0.25$ (inelastic)

$P=20$ $Q=300 \Rightarrow E = -2/3$ (inelastic)

$P=30$ $Q=200 \Rightarrow E = -1.5$ (elastic)

$P=25$ $Q=250 \Rightarrow E = -1$ (unit elastic)

⑤ To estimate the model, use a simple supply/demand framework.

(a) $\left. \begin{array}{l} Q_D = a + bP \\ Q_S = c + dP \end{array} \right\}$ use the data above (elasticity for the general population)
to find the parameters a, b, c , and d .

• $E_D = \frac{\partial Q_D}{\partial P} \cdot \frac{P}{Q_D}$ and $\partial Q_D / \partial P = b$

So; $b = E_D \cdot \frac{Q}{P} = -0.45 \left(\frac{17.4}{5.51} \right) = -1.42$

• $17.4 = a - 1.42 \times 5.51 \Rightarrow a = 25.2$

$Q_D = 25.2 - 1.42P$

• $E_S = \frac{\partial Q_S}{\partial P} \cdot \frac{P}{Q_S}$ and $\partial Q_S / \partial P = d$

$d = E_S \cdot \frac{Q_S}{P} = 7 \cdot \left(\frac{17.4}{5.51} \right) = 22.1$

• $17.4 = c + 22.1 \times 5.51 \Rightarrow c = -104.4$

$Q_S = -104.4 + 22.1P$

(b) We already know eqm. price and quantity, but we should double check.

$$25.2 - 1.42P = -104.4 + 22.1P$$

$$129.6 = 23.52P$$

$$P = 5.51$$

$$Q = 25.2 - 1.42 \times 5.51 = 17.4$$

(c) To put in the effect of the tax, we need to do the following:

• We have $Q_s = c + d \cdot P$

Solve for the price: $P = \left(\frac{1}{d}\right) Q_s - \frac{c}{d}$

Add the tax: $P = \frac{1}{d} \cdot Q_s - \frac{c}{d} + \1

Now, solve for quantity: $Q_s = (c - d) + dP$

New supply curve is: $Q_s = -126.5 + 22.1P$

OR: $Q_s = -104.4 + 22.1P$

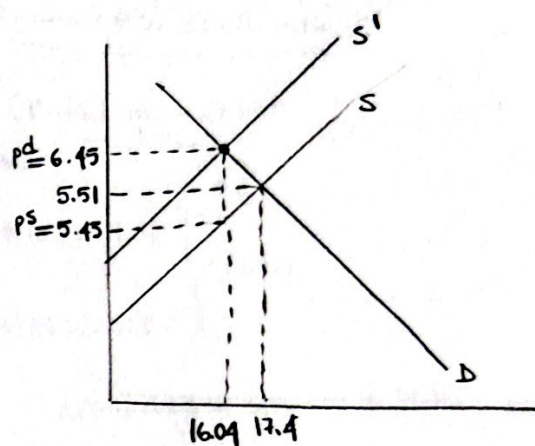
Now suppliers receive less, so the new supply:

$$Q_s' = -104.4 + 22.1(P-1)$$

$$= -126.5 + 22.1P$$

(d) We need to use new supply curve and solve for the new eqm. price and quantity:

$$\left. \begin{aligned} Q_d &= 25.2 - 1.42P \\ Q_s &= -126.5 + 22.1P \end{aligned} \right\} \begin{aligned} 25.2 - 1.42P &= -126.5 + 22.1P \\ 151.7 &= 23.52P \\ P &= 6.45 \\ Q &= 25.2 - 1.42 \times 6.45 \\ &= 16.04 \end{aligned}$$



• Note that the price does not rise by the full dollar. Cigarette suppliers absorb some of the tax by dropping their price a little. Cigarette sales fall by 6% from 17.48 to 16.04.

(e) Given a 16% rise in price from \$5.51 to \$6.45 and an elasticity of demand for teenagers of -0.7, sales fall by $-0.7 \times 16 = -11\%$.

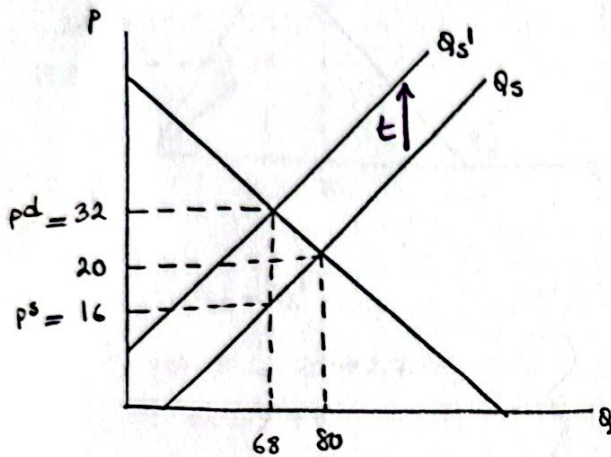
⑥ $Q_D = 100 - P$ and $Q_S = 20 + 3P$

Find the eqm. first: $100 - P = 20 + 3P$

$$80 = 4P \Rightarrow P^* = 20$$

$$Q = 100 - P \Rightarrow Q^* = 80$$

(a) Unit tax of \$16 on supplies:



① $P^{\text{new}} = P^{\text{old}} + \text{tax}$

$$P^{\text{new}} = \frac{Q_S - 20}{3} + 16 = \frac{Q_S + 28}{3}$$

$$Q_S' = -28 + 3P$$

$$Q_S' = Q_D \Rightarrow -28 + 3P = 100 - P$$

$$4P = 128$$

$$P^d = 32 \text{ and } Q = 68$$

$$P^s = 32 - 16 = 16$$

or

② $Q_S' = 20 + 3(P - 16) = -28 + 3P$

and then solve

$$\begin{aligned} |\Delta CS| &= (32 - 20) \times 68 + (32 - 20) \times (80 - 68) \times \frac{1}{2} = 816 + 108 = 924 \\ |\Delta PS| &= (20 - 16) \times 68 + (20 - 16) \times (80 - 68) \times \frac{1}{2} = 272 + 36 = 308 \end{aligned} \quad \left. \vphantom{\begin{aligned} |\Delta CS| &= (32 - 20) \times 68 + (32 - 20) \times (80 - 68) \times \frac{1}{2} = 816 + 108 = 924 \\ |\Delta PS| &= (20 - 16) \times 68 + (20 - 16) \times (80 - 68) \times \frac{1}{2} = 272 + 36 = 308 \end{aligned}} \right\} 1232$$

$$\begin{aligned} \text{Tax Revenue} &= (32 - 16) \times 68 = 1088 \\ \text{DWL} &= (32 - 16) \times (80 - 68) \times \frac{1}{2} = 144 \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{Tax Revenue} &= (32 - 16) \times 68 = 1088 \\ \text{DWL} &= (32 - 16) \times (80 - 68) \times \frac{1}{2} = 144 \end{aligned}} \right\} 1232$$

$$\Delta PS + \Delta CS = \text{Tax Rev} + \text{DWL}$$

$$\text{Tax Burden on consumers} = (32 - 20) \times 68 = 816$$

$$\text{Tax Burden on producers} = (20 - 16) \times 68 = 272$$

(b) Left as an exercise. Note that this time demand eqn. will change and shift.

(c) Ad-valorem tax of 10% on supplier:

① $P^{\text{new}} = 1.1 P^{\text{old}}$

$Q_s = 20 + 3P$ and $P = \frac{Q_s - 20}{3}$

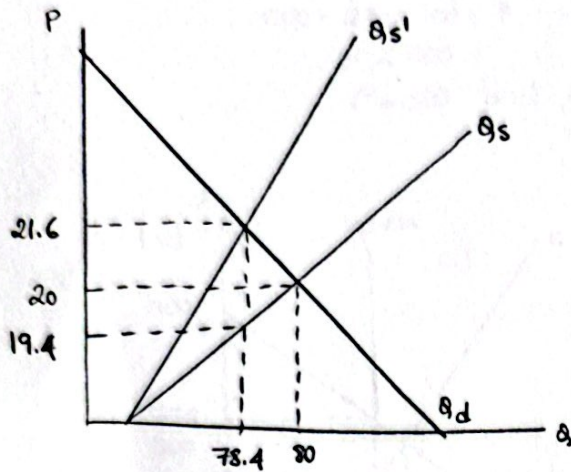
$P = \frac{Q_s - 20}{3} \cdot \frac{11}{10}$

$30P = 11Q_s - 220$

$Q'_s = 20 + \frac{30}{11} P \approx 20 + 2.7P$

or

② $Q'_s = 20 + 3(1 - 0.1)P = 20 + 2.7P$



$Q_d = Q'_s$

$100 - P = 20 + 2.7P$

$80 = 3.7P$

$P^d \approx 21.6 \quad Q \approx 78.4$

$P^s \approx 19.4$

$|\Delta CS| = (21.6 - 20) \times 78.4 + (21.6 - 20) \times (80 - 78.4) \times \frac{1}{2}$

$|\Delta PS| = (20 - 19.4) \times 78.4 + (20 - 19.4) \times (80 - 78.4) \times \frac{1}{2}$

Tax Revenue = $(21.6 - 19.4) \times 78.4$

DWL = $(80 - 78.4) (21.6 - 19.4) \times \frac{1}{2}$

Tax burden on consumer = $(21.6 - 20) \times 78.4$

Tax burden on producer = $(20 - 19.4) \times 78.4$

(d) left as an exercise. This time the demand curve will rotate instead.

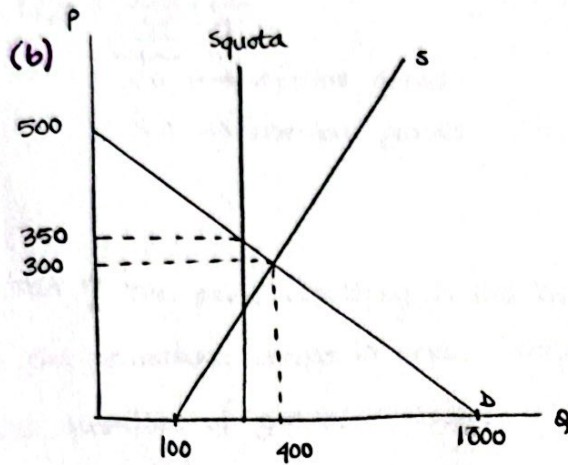
⑦

(a) The eqm. solution with no govt intervention is:

$$1000 - 2P = 100 + P$$

$$3P = 900$$

$$P^* = 300 \text{ and } Q^* = 400$$



• When quota is imposed at 300 units, supply cannot exceed that level, regardless of price. Thus, supply curve becomes vertical at 300 units. The new eqm. quantity is 300 and price is determined by where the supply curve with the quota (Squota) intersects the demand curve.

$$1000 - 2P = 300$$

$$P = 350$$

⑧

(a) TRUE! A price elasticity of less than 1 (in absolute terms) means that demand is not very sensitive to price, while an elasticity greater than 1 means that demand is increasingly sensitive to price.

$$TR = Q \times P$$

$$\Delta TR = \Delta Q \times \Delta P$$

$$\begin{matrix} \uparrow & \downarrow & \uparrow \end{matrix}$$

$$\epsilon_{D,P} < 1 \Rightarrow \% \Delta Q < \% \Delta P$$

(b) FALSE! If total revenue increases by 10% when the price increases by 10%, then it means that quantity stays the same. However, unitary elastic demand says that a 10% increase in price causes 10% decrease in quantity.

$$\Delta TR = \Delta Q \times \Delta P \text{ and } \epsilon_{D,P} = 1 \Rightarrow \% \Delta Q = \% \Delta P$$

$$\begin{matrix} \uparrow & - & \uparrow \end{matrix}$$

(c) FALSE! Drug interdiction is a supply side shock, so it decreases the supply resulting in an increase in price.

⑧

- (d) FALSE! A negative income elasticity of demand is linked with inferior goods, meaning rising incomes will lead to a drop in demand and may mean shift to luxury goods. A positive income elasticity of demand is linked with normal goods. In this case, a rise in income will lead to a rise in demand.

$$\epsilon_{D,I} = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta I}{I}}$$

$< 0 \rightarrow$ inferior goods
 $> 0 \rightarrow$ normal goods

- (e) TRUE! The price elasticity is the ratio of the percentage change in quantity to the percentage change in price. steepness / flatness is related to actual prices and quantities of goods.

⑨

$$P_C - P_S = t$$

$$dP_C - dP_S = dt$$

$$dP_S = dP_C - dt$$

To maintain eqm: $dQ^D = dQ^S$

$$\frac{dQ^D}{dP_C} \cdot dP_C = \frac{dQ^S}{dP_S} \cdot (dP_C - dt)$$

$$\frac{dQ^D}{dP_C} \cdot dP_C = \frac{dQ^S}{dP_S} (dP_C - dt)$$

$$\frac{1}{dt} \left[\frac{dQ^D}{dP_C} \cdot dP_C = \frac{dQ^S}{dP_S} \cdot dP_C - \frac{dQ^S}{dP_S} \cdot dt \right] \frac{1}{dt}$$

$$\frac{dQ^D}{dP_C} \cdot \frac{dP_C}{dt} = \frac{dQ^S}{dP_S} \cdot \frac{dP_C}{dt} - \frac{dQ^S}{dP_S}$$

$$\frac{dQ^S}{dP_S} = \frac{dQ^S}{dP_S} \cdot \frac{dP_C}{dt} - \frac{dQ^D}{dP_C} \cdot \frac{dP_C}{dt}$$

$$\frac{dQ^S}{dP_S} = \frac{dP_C}{dt} \left[\frac{dQ^S}{dP_S} - \frac{dQ^D}{dP_C} \right]$$

$$Q^D = f(P_C)$$

$$dQ^D = \frac{dQ^D}{dP_C} \cdot dP_C \text{ and}$$

$$Q^S = f(P_S)$$

$$dQ^S = \frac{dQ^S}{dP_S} \cdot dP_S$$

$$\frac{dP_C}{dt} = \frac{\frac{dQ^S}{dP_S}}{\frac{dQ^S}{dP_S} - \frac{dQ^D}{dP_C}} \times \frac{P}{Q}$$

$$\frac{dP_C}{dt} = \frac{\epsilon_S}{\epsilon_S - \epsilon_D}$$

⑨

⑩ We have linear supply and demand:

$$\left. \begin{array}{l} Q_d = a + bP \\ Q_s = c + dP \end{array} \right\} \begin{array}{l} a + bP = c + dP \\ a - c = P(d - b) \\ P = \frac{a - c}{d - b} \end{array}$$

(i) tax is levied on producers:

$$Q_s' = c + (d - t)P$$

$$c + (d - t)P = a + bP$$

$$a - c = (d - t - b)P$$

$$P_b = \frac{a - c}{d - b - t}$$

$$P_s = \frac{a - c}{d - b - t} - t$$

(ii) tax is levied on consumers

$$Q_d' = a + (b + t)P$$

$$a + (b + t)P = c + dP$$

$$a - c = (d - b - t)P$$

$$P_s = \frac{a - c}{d - b - t}$$

$$P_b = \frac{a - c}{d - b - t} + t$$

Price does not increase by the full amount of the tax. Although tax is levied on producers, burden of the tax is shared by both producers and consumers depending on elasticities.

⑪ $Q_s = 10P_J - 5P_A$

$$Q_D = 100 - 15P_J + 10P_T$$

$$P_A = 1 \text{ and } P_T = 5$$

(a) $10P_J - 5.1 = 100 - 15P_J + 10.5$

$$10P_J - 5 = 150 - 15P_J$$

$$25P_J = 155$$

$$P_J = 6.2 \text{ and } Q = 10 \times 6.2 - 5.1 = 57$$

(b) If P_A increases to P_A' , then;

$$10P_J - 5P_A' = 100 - 15P_J + 10 \times 5$$

$$25P_J = 150 + 5P_A'$$

$$P_J = \frac{150 + 5P_A'}{25}$$

$$\begin{aligned} \text{and } Q_s &= 10P_J - 5P_A' \\ &= \frac{150 + 5P_A'}{25} \times 10 - 5P_A' \\ &= \frac{300 + 10P_A' - 25P_A'}{5} = \frac{300 - 15P_A'}{5} \end{aligned}$$

$Q_s \downarrow$ as $P_A \uparrow$
(supply shifts to the left)

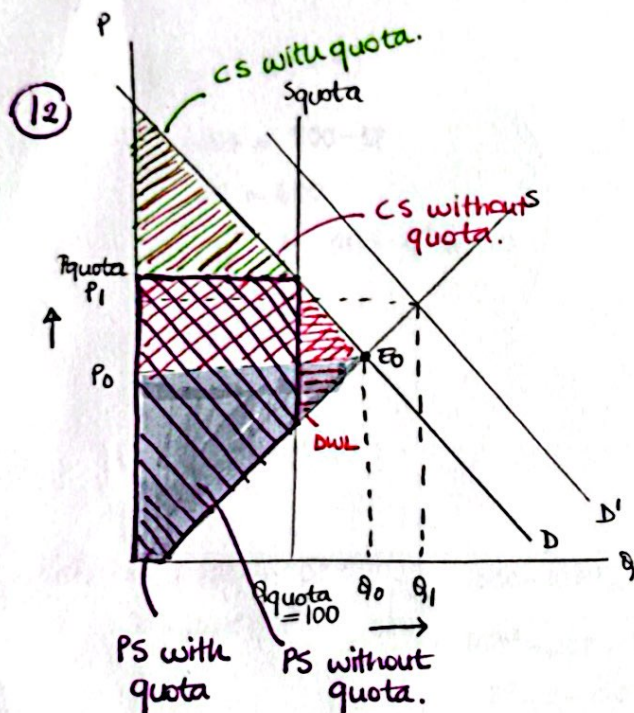
$$(c) \quad 10P_J - 5.1 = 100 - 15P_J + 10P_T'$$

$$25P_J = 105 + 10P_T'$$

$$P_J = \frac{21 + 2P_T'}{5} \quad \text{and} \quad Q_d = 100 - \frac{3}{5} \cdot \frac{21 + 2P_T'}{5} + 10P_T'$$

$$Q_d = 100 - 63 - 6P_T' + 10P_T'$$

$$Q_d = 37 + 4P_T' \quad (Q_d \uparrow \text{ as } P_T \uparrow \text{ — shifts to the right})$$



⑬ $P = 12 - 0.5Q_D$ and $P = 0.1Q_S$

$$12 - 0.5Q = 0.1Q$$

$$12 = 0.6Q$$

$$Q^* = 20 \text{ and } P^* = 2$$

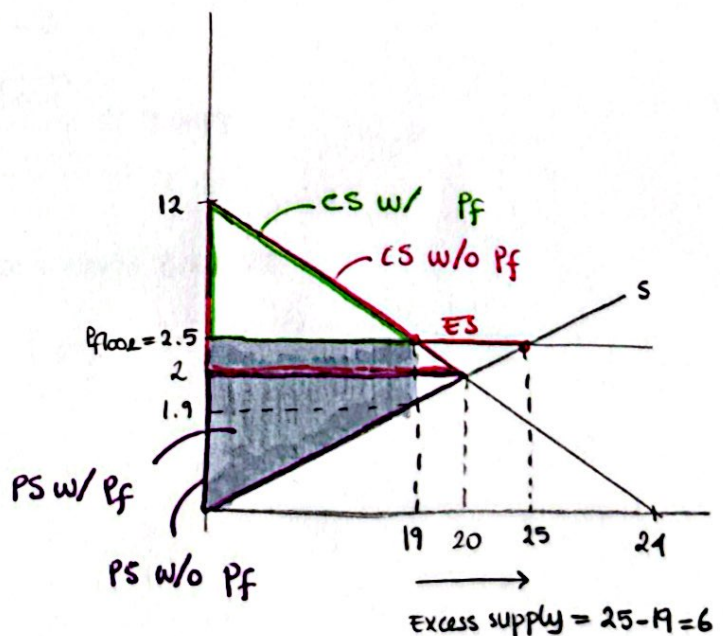
At price floor $P_f = 2.5$

$$Q_D = 19, \quad Q_S = 25$$

Without price floor:

$$CS = (12 - 2) \times 20 \times \frac{1}{2} = 100$$

$$PS = 2 \times 20 \times \frac{1}{2} = 20$$



With price floor:

$$CS = (12 - 2.5) \times 19 \times \frac{1}{2} = 90.25$$

$$PS = (2.5 - 1.9) \times 19 + 1.9 \times 19 \times \frac{1}{2}$$

$$= 11.4 + 18.05 = 29.45$$

14) $Q_D = 500 - 5P$

$Q_S = 20P$

(a) $500 - 5P = 20P$

$25P = 500$

$P^* = 20$ and $Q^* = 400$

(b) $t = 5$

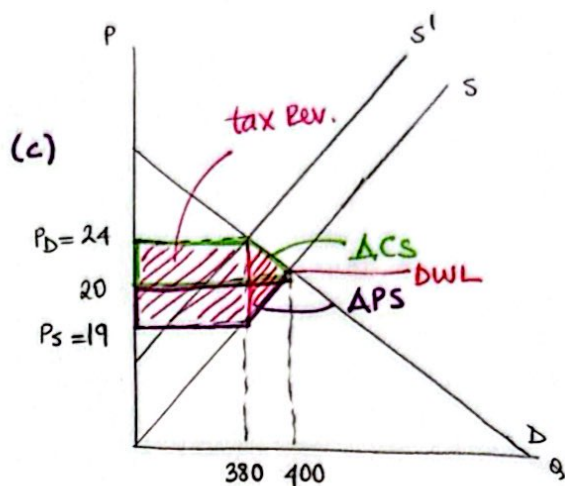
$Q_{S'} = 20(P - 5)$

$20P - 100 = 500 - 5P$

$25P = 600$

$P_D^* = 24$ and $Q^* = 380$

$P_S^* = 19$



$\Delta CS = (24 - 20) \times 380 + 4 \times 20 \times \frac{1}{2}$

$\Delta PS = (20 - 19) \times 380 + 1 \times 20 \times \frac{1}{2}$

Tax Rev. = 5×380

DWL = $5 \times 20 \times \frac{1}{2}$

(d) $Q_D = 1000 - 10P^2$
 $Q_S = 20P$

$1000 - 10P^2 = 20P$

$10P^2 - 20P - 1000 = 0$

$P^2 - 2P - 100 = 0$

$P^* = \frac{-2 + \sqrt{404}}{2} \approx 9.0499$

and

$Q^* = 20P = 20 \times 9.0499 \approx 180.998$