

PROBLEM SET II SOLUTIONS

Section I: Chapter 4

Question 2

Suppose household preferences are described by the utility function

$$U(C, \ell) = 2\sqrt{C} + \alpha\ell$$

where C stands for consumption of market goods and ℓ stands for leisure.

Assuming the market (real) wage is w and the total amount of time available is h , derive expressions for the household's consumption and labor supply decisions as a function of w and h . (For simplicity, assume the household has no nonmarket income). Does the income effect ever dominate the substitution effect? How would things change if the household had some nonmarket income? Explain intuitively.

Solution:

The household's optimality condition is

$$\frac{U_\ell}{U_C} = w$$

Computing the derivatives and substituting in gives you

$$\alpha\sqrt{C} = w \Rightarrow C = \left(\frac{w}{\alpha}\right)^2$$

The household's budget constraint can be written

$$C + w \cdot \ell = w \cdot h$$

Substituting in the above expression for C and simplifying gives

$$\frac{w}{\alpha^2} + \ell = h \Rightarrow N^s = h - \ell = \frac{w}{\alpha^2}$$

Notice a couple of funny things here: (1) Consumption only depends on the wage rate! It doesn't depend on either leisure, or the total time endowment, (2) Labor supply only depends on the wage too, and is also independent of the total time endowment. In other words, if God gave this person an extra 10 hours a day, she would use it all up as leisure. She wouldn't work an extra minute (assuming of course the wage stays the same). Why is this? What's going on here? Notice that preferences here are linear in ℓ . The marginal utility of leisure is constant, not diminishing. (These kinds of preferences are called 'quasi-linear'). As a result, this

person/household will consume all extra income (beyond a certain point) in the form of leisure. The income effect on consumption is zero! Geometrically, as the budget constraint shifts out in a parallel fashion, the tangency points all occur at the same level of consumption! Obviously then, without having to do any math, we know that if this person gets extra non-market income, consumption and labor supply will remain unchanged.

Section II: Chapter 5

Question 8

In the 1930s Keynes famously argued that the best way to stimulate employment was to pay people to dig holes and fill them up again. This question explores the logic behind that advice.

- a) Suppose households have the following preferences:

$$U(C, l, G) = \sqrt{C + G} + \sqrt{l}$$

where, as before, C is market consumption and l is leisure. The new element here is government purchases, G . Note that the above preferences imply that government-provided goods are perfect substitutes for private consumption. Of course, the government must pay for the goods it buys and suppose it does so by imposing lump-sum taxes, T , on the household. Since there is only one period, the government budget constraint is just $C = G$. Now the household's budget constraint is:

$$C + w \cdot l = w - T = w - G$$

For simplicity, suppose that the production takes the simple form:

$$Y = N = 1 - l$$

so that the marginal product of labor (and therefore the wage) is constant and equal to one. Given this information, solve for the household's labor supply curve. Does it depend on G ? Why or why not? What happens to output, employment, and consumption when G increases?

Solution:

Once again, the household's optimality condition is

$$\frac{U_l}{U_c} = w$$

Taking the derivatives (using the chain rule) gives

$$\sqrt{\frac{C + G}{l}} = w \quad \Rightarrow \quad \frac{C + G}{l} = w^2$$

The budget constraint now is a little different, since the household must pay taxes to finance government spending. It now becomes

$$C + w \cdot \ell = w \cdot h - T$$

In this simple one-period model the government budget constraint implies $G = T$. Substituting this along with the household's optimality condition into the budget constraint gives

$$C + G = w(h - \ell) = wN^s \Rightarrow N^s = w\ell \Rightarrow N^s = \frac{w}{1 + w}h$$

Hence, the household works a fraction of its total available time, where the fraction is increasing in w . However, notice that labor supply does not depend on G . Neither does labor demand, since the marginal product of labor is constant (and equal to one). Therefore, output and employment do not depend on fiscal policy. This is because government-supplied services are perfect substitutes for private consumption. When the government raises taxes to buy things, the private sector just lowers its consumption one-for-one. There is no adverse wealth effect from higher taxes, since the government is buying things, the household would have bought anyway!

- b) Now suppose the government buys useless things (e.g., holes in the ground), so that the household's utility function is as follows:

$$U(C, \ell) = \sqrt{C} + \sqrt{\ell}$$

In other words, government purchases do not provide utility to the household. Does labor supply now depend on G ? Why or why not? (Hint: Your answer should refer to income and substitution effects.) If G increases, how does output and employment respond? Was Keynes right? (WARNING: This is a neoclassical economy. There is no unemployment! Keynes had in mind a different sort of economy, but this example sheds some light on his thinking.)

Solution:

We just need to repeat the above analysis, with the new utility function. The household's optimality condition becomes

$$\frac{C}{\ell} = w^2$$

Substituting this into the budget constraint now yields

$$N^s = \frac{w}{1 + w}h + \frac{G}{w(1 + w)}$$

Notice that now when G increases, labor supply increases. Why the difference? Now the government is buying things people don't care about (ie, digging holes). Raising taxes to pay for these things represents a negative wealth effect, which makes people work harder. There is

no offsetting substitution effect here since wages are pinned down by the constant marginal product of labor. Therefore, Keynes was right! If your goal is to stimulate employment, then have the government buy useless things! (Of course, in this economy, where there are no distortions, doing this makes people worse off! There is an important difference between output and welfare). The economy Keynes had in mind was subject to monopoly distortions, where output is demand-determined, at least in the short-run. That is, Keynes was more concerned with the demand-side than the supply-side. If output is sub-optimally low, then stimulating aggregate demand is useful. Doing this is more effective when the government buys useless things, since it doesn't crowd out private spending. We'll discuss this more later on.