

PROBLEM SET

1. Suppose you are given the following:

Production functions: $X = K_x^{1/3} L_x^{2/3}$
 $Y = K_y^{2/3} L_y^{1/3}$

Utility functions: $U^1 = X_1 Y_1$
 $U^2 = X_2 Y_2$

Welfare function: $W = U^1 U^2$

Endowments: $L^1 = \frac{1}{3} \bar{L}$
 $L^2 = \frac{2}{3} \bar{L}$
 $\bar{K}$

In a competitive economy, what proportion of the capital stock $\bar{K}$ should be owned by Person 1 and Person 2 if social welfare to be maximized?

For the competitive equilibrium to be socially optimal, the distribution of factor ownership must be such that each consumer can (and will) buy the bundle which corresponds to the welfare maximizing allocation of the economy.

1. Condition for social optimum:

$$\frac{\frac{\partial U^2}{\partial X_2}}{\frac{\partial U^1}{\partial X_1}} = \frac{\frac{\partial W}{\partial U^1}}{\frac{\partial W}{\partial U^2}}$$

$$\frac{Y_2}{Y_1} = \frac{U^2}{U^1}$$

Substituting $\frac{X_2 Y_2}{X_1 Y_1}$ for $\frac{U^2}{U^1}$ gives us $X_1 = X_2$(1)

Similarly, $Y_1 = Y_2$(2)

From (1) and (2), social optimum requires that consumers get exactly half of the total production of each of the two goods.

2. Competitive Equilibrium Conditions:

(a) Utility maximization (consumers are identical)

$$\text{Maximize} \quad U^1 = X_1 Y_1$$

$$\text{Subject to} \quad P_x X_1 + P_y Y_1 = wL^1 + rK^1 + \theta_{x^1} \pi_x + \theta_{y^1} \pi_y$$

$$L = X_1 Y_1 + \lambda \left[P_x X_1 + P_y Y_1 - wL^1 - rK^1 - \theta_{x^1} \pi_x - \theta_{y^1} \pi_y \right]$$

$$\begin{aligned} \frac{\partial L}{\partial X_1} &= Y_1 + \lambda P_x = 0 \\ \frac{\partial L}{\partial Y_1} &= X_1 + \lambda P_y = 0 \end{aligned} \Rightarrow \frac{Y_1}{X_1} = \frac{-\lambda P_x}{-\lambda P_y} \Rightarrow \boxed{P_y Y_1 = P_x X_1}$$

$$\frac{\partial L}{\partial \lambda} = P_x X_1 + P_y Y_1 - wL^1 - rK^1 - \theta_{x^1} \pi_x - \theta_{y^1} \pi_y = 0$$

$$2P_y Y_1 = wL^1 + rK^1 + \theta_{x^1} \pi_x + \theta_{y^1} \pi_y$$

$$\boxed{Y_1 P_y = \frac{wL^1 + rK^1 + \theta_{x^1} \pi_x + \theta_{y^1} \pi_y}{2}}$$

Similarly for $X_1 P_x$:

$$\boxed{X_1 P_x = \frac{wL^1 + rK^1 + \theta_{x^1} \pi_x + \theta_{y^1} \pi_y}{2}}$$

$$\text{Maximize} \quad U^2 = X_2 Y_2$$

$$\text{Subject to} \quad P_x X_2 + P_y Y_2 = wL^2 + rK^2 + \theta_{x^2} \pi_x + \theta_{y^2} \pi_y$$

$$L = X_2 Y_2 + \lambda \left[P_x X_2 + P_y Y_2 - wL^2 - rK^2 - \theta_{x^2} \pi_x - \theta_{y^2} \pi_y \right]$$

$$\begin{aligned} \frac{\partial L}{\partial X_2} &= Y_2 + \lambda P_x = 0 \\ \frac{\partial L}{\partial Y_2} &= X_2 + \lambda P_y = 0 \end{aligned} \Rightarrow \frac{Y_2}{X_2} = \frac{-\lambda P_x}{-\lambda P_y} \Rightarrow \boxed{Y_2 P_y = X_2 P_x}$$

$$\frac{\partial L}{\partial \lambda} = P_x X_2 + P_y Y_2 - wL^2 - rK^2 - \theta_{x^2} \pi_x - \theta_{y^2} \pi_y = 0$$

$$2P_x X_2 = wL^2 + rK^2 + \theta_{x^2} \pi_x + \theta_{y^2} \pi_y$$

$$P_x X_2 = \frac{wL^2 + rK^2 + \theta_{x^2} \pi_x + \theta_{y^2} \pi_y}{2}$$

Similarly for $P_y Y_2$:

$$P_y Y_2 = \frac{wL^2 + rK^2 + \theta_{x^2} \pi_x + \theta_{y^2} \pi_y}{2}$$

Having $X = K^{1/3} L^{2/3}$ degree of homogeneity should be checked.

$$f(\lambda K, \lambda L) = (\lambda K)^{1/3} (\lambda L)^{2/3} = \lambda^{1/3+2/3} (K^{1/3} L^{2/3}) = \lambda^1 (K^{1/3} L^{2/3})$$

$$\lambda^k = \lambda^1 \Rightarrow k=1 \Rightarrow \text{CRTS} \Rightarrow \pi_x = 0$$

Similarly, for $Y = K^{2/3} L^{1/3}$ we have CRTS implying $\pi_y = 0$.

We can write:

$$X_1 P_x = \frac{wL^1 + rK^1}{2} = Y_1 P_y$$

$$X_2 P_x = \frac{wL^2 + rK^2}{2} = Y_2 P_y$$

$$X_i P_x = \frac{wL^i + rK^i}{2} = Y_i P_y \dots\dots\dots(3)$$

Utility maximization implies that both consumers spend $\frac{1}{2}$ of their income on good X and $\frac{1}{2}$ on good Y.

$$P_x X_1 + P_y Y_1 + P_x X_2 + P_y Y_2 = P_x X + P_y Y$$

||

$$wL^1 + rK^1 + wL^2 + rK^2 = wL_x + rK_x + wL_y + rK_y$$

(b) Profit maximization

Industry X:

$$\text{Maximize} \quad \pi_x = P_x X - wL_x - rK_x$$

$$\text{Subject to} \quad X = K_x^{1/3} L_x^{2/3}$$

$$\text{Maximize} \quad \pi_x = P_x K_x^{1/3} L_x^{2/3} - wL_x - rK_x$$

$$\frac{\partial \pi_x}{\partial L_x} = \frac{2}{3} P_x L_x^{-1/3} K_x^{1/3} - w = 0$$

$$\frac{\partial \pi_x}{\partial K_x} = \frac{1}{3} P_x K_x^{-2/3} L_x^{2/3} - r = 0$$

$$\frac{\frac{2}{3} \frac{P_x K_x^{1/3}}{L_x^{1/3}}}{\frac{1}{3} \frac{P_x L_x^{2/3}}{K_x^{2/3}}} = \frac{w}{r}$$

$$\frac{2K_x}{L_x} = \frac{w}{r} \Rightarrow 2rK_x = wL_x \Rightarrow rK_x = \frac{wL_x}{2}$$

Note:

$$TC_x = wL_x + rK_x$$

$$TC_x = wL_x + \frac{wL_x}{2}$$

$$2TC_x = 2wL_x + wL_x$$

$$2TC_x = 3wL_x$$

$$TC_x = rK_x + 2rK_x$$

$$TC_x = 3rK_x$$

$$\boxed{wL_x = \frac{2}{3} TC_x}$$

$$\boxed{rK_x = \frac{1}{3} TC_x}$$

Factor payments in the industry X: 1/3 goes to K and 2/3 goes to L.

Industry Y:

$$\text{Maximize } \pi_y = P_y K_y^{2/3} L_y^{1/3} - wL_y - rK_y$$

$$\frac{\partial \pi_y}{\partial L_y} = \frac{1}{3} P_y L_y^{-2/3} K_y^{2/3} - w = 0$$

$$\frac{\partial \pi_y}{\partial K_y} = \frac{2}{3} P_y K_y^{-1/3} L_y^{1/3} - r = 0$$

$$\frac{\frac{1}{3} \frac{P_y K_y^{2/3}}{L_y^{2/3}}}{\frac{2}{3} \frac{P_y L_y^{1/3}}{K_y^{1/3}}} = \frac{w}{r}$$

$$\frac{K_y}{2L_y} = \frac{w}{r} \Rightarrow rK_y = 2wL_y$$

Note:

$$TC_y = wL_y + rK_y$$

$$TC_y = wL_y + 2wL_y$$

$$TC_y = 3wL_y$$

$$TC_y = rK_y + \frac{rK_y}{2}$$

$$2TC_y = 3rK_y$$

$$\boxed{wL_y = \frac{1}{3} TC_y}$$

$$\boxed{rK_y = \frac{2}{3} TC_y}$$

Factor payments in the industry Y: 1/3 goes to L and 2/3 goes to K.

Note:

$$\text{By } \pi_x = 0 \quad P_x \sum_{i=1}^2 X_i = TC_x \quad \text{where } P_x \sum_{i=1}^2 X_i \text{ is } TR_x.$$

$$P_x X_i = \frac{wL^i + rK^i}{2} = P_y Y_i$$

$$P_x X_1 = \frac{wL^1 + rK^1}{2}$$

$$P_x X_2 = \frac{wL^2 + rK^2}{2}$$

$$P_x (X_1 + X_2) = \frac{w(L^1 + L^2) + r(K^1 + K^2)}{2}$$

$$\text{Therefore, } P_x \sum_{i=1}^2 X_i = \frac{w\bar{L} + r\bar{K}}{2} = TC_x$$

$$\text{By } \pi_y = 0 \quad P_y \sum_{i=1}^2 Y_i = TC_y \quad \text{where } P_y \sum_{i=1}^2 Y_i \text{ is } TR_y.$$

$$\text{Similarly, } P_y \sum_{i=1}^2 Y_i = \frac{w\bar{L} + r\bar{K}}{2} = TC_y$$

$$wL_x = \frac{2}{3} TC_x = \frac{2}{3} \left[\frac{w\bar{L} + r\bar{K}}{2} \right] = \frac{2}{6} [w\bar{L} + r\bar{K}]$$

$$rK_x = \frac{1}{3} TC_x = \frac{1}{3} \left[\frac{w\bar{L} + r\bar{K}}{2} \right] = \frac{1}{6} [w\bar{L} + r\bar{K}]$$

$$wL_y = \frac{1}{3} TC_y = \frac{1}{3} \left[\frac{w\bar{L} + r\bar{K}}{2} \right] = \frac{1}{6} [w\bar{L} + r\bar{K}]$$

$$rK_y = \frac{2}{3} TC_y = \frac{2}{3} \left[\frac{w\bar{L} + r\bar{K}}{2} \right] = \frac{2}{6} [w\bar{L} + r\bar{K}]$$

$$wL_x + wL_y = w [L_x + L_y] = \left[\frac{2}{6} + \frac{1}{6} \right] [w\bar{L} + r\bar{K}]$$

$$w\bar{L} = \frac{3}{6}(w\bar{L} + r\bar{K}) = \frac{w\bar{L} + r\bar{K}}{2}$$

$$rK_x + rK_y = r[K_x + K_y] = \left[\frac{1}{6} + \frac{2}{6} \right] [w\bar{L} + r\bar{K}]$$

$$r\bar{K} = \frac{3}{6}(w\bar{L} + r\bar{K}) = \frac{w\bar{L} + r\bar{K}}{2}$$

$$\text{Therefore, } r\bar{K} = w\bar{L} = \frac{w\bar{L} + r\bar{K}}{2} \dots\dots\dots(4)$$

½ of total factor payments goes to K and ½ goes to L.

Summary:

$$(1) \text{ and } (2) \quad \begin{matrix} X_2 = X_1 \\ Y_2 = Y_1 \end{matrix} \quad \text{for social optimum}$$

$$(3) \quad P_x X_i = \frac{wL^i + rK^i}{2} = P_y Y_i \quad \text{for utility maximization}$$

$$(4) \quad r\bar{K} = w\bar{L} = \frac{w\bar{L} + r\bar{K}}{2} \quad \text{for profit maximization (market clearing)}$$

By (1), (2) and (3):

$$P_x X_i + P_y Y_i = \frac{P_x \sum X_i + P_y \sum Y_i}{2} = wL^i + rK^i \dots\dots\dots(5)$$

By (4):

$$r\bar{K} = w\bar{L} = \frac{r\bar{K} + w\bar{L}}{2} = \frac{P_x \sum X_i + P_y \sum Y_i}{2}$$

We know that from (5): $P_x X_1 + P_y Y_1 = wL^1 + rK^1$ (share of consumer 1 in total income)

$P_x X_2 + P_y Y_2 = wL^2 + rK^2$ (share of consumer 2 in total income)

As $X_2 = X_1$ and $Y_2 = Y_1$, each consumer has ½ of total income.

Since, $L^1 = \frac{1}{3}\bar{L}$

$$wL^1 = w \frac{1}{3}\bar{L}$$

$$wL^1 = \frac{1}{3} \frac{P_x \sum X_i + P_y \sum Y_i}{2} = \frac{1}{6} (P_x \sum X_i + P_y \sum Y_i)$$

$$P_x X_1 + P_y Y_1 = wL^1 + rK^1$$

$$\frac{1}{2}(P_x \sum X_i + P_y \sum Y_i) = \frac{1}{6}(P_x \sum X_i + P_y \sum Y_i) + rK^1$$

$$rK^1 = \frac{1}{2}(P_x \sum X_i + P_y \sum Y_i) - \frac{1}{6}(P_x \sum X_i + P_y \sum Y_i)$$

$$rK^1 = \frac{2}{6}(P_x \sum X_i + P_y \sum Y_i)$$

$$rK^1 = \frac{2}{3} \frac{(P_x \sum X_i + P_y \sum Y_i)}{2}$$

$$\text{where } \frac{(P_x \sum X_i + P_y \sum Y_i)}{2} = r\bar{K}$$

$$rK^1 = \frac{2}{3} r\bar{K}$$

$$\text{Therefore, } K^1 = \frac{2}{3} \bar{K} \text{ and } K^2 = \frac{1}{3} \bar{K}$$

Note 1: Second order conditions must hold.

Note 2: Optimum allocation of L and K in between X and Y production is as follows:

$$K_x = \frac{1}{3} \bar{K}$$

$$K_y = \frac{2}{3} \bar{K}$$

$$L_x = \frac{2}{3} \bar{L}$$

$$L_y = \frac{1}{3} \bar{L}$$

2. Suppose two individuals (A and B) each have 10 hours of labor to devote to producing either X or Y.

A's utility function is given by:

$$U^A = X^{0.3}Y^{0.7}$$

whereas B's utility function is given by:

$$U^B = X^{0.5}Y^{0.5}$$

The individuals do not care whether they produce X or Y, and the production function for each good is given by:

$$X = 2L$$

$$Y = 3L$$

where L is the total labor devoted to production of each good.

- What must the price ratio $\frac{P_x}{P_y}$ be?
- Given this price ratio, how much X and Y will A and B demand? (Hint: set the wage equal to one here.)
- How should labor be allocated between X and Y to satisfy the demand calculated in part (b)?

Competitive equilibrium requires:

- Profit maximization in sectors X and Y.
- Utility maximization by A and B.
- Clearing of three markets.

a)

Profit maximization in X-industry:

$$P_x \frac{\partial X}{\partial L_x} = w$$

$$P_x 2 = w$$

Profit maximization in Y-industry:

$$P_y \frac{\partial Y}{\partial L_y} = w$$

$$P_y 3 = w$$

$$2P_x = 3P_y$$

$$\boxed{\frac{P_x}{P_y} = \frac{3}{2}}$$

b)

Utility maximization by A:

$$W = 1$$

$$P_x 2 = 1 \quad \Rightarrow \quad P_x = \frac{1}{2}$$

$$P_y 3 = 1 \quad \Rightarrow \quad P_y = \frac{1}{3}$$

$$\text{Maximize } U_A = X_A^{0.3} Y_A^{0.7}$$

$$\text{Subject to } \frac{1}{2} X_A + \frac{1}{3} Y_A = 1(10)$$

$$\frac{\partial U_A}{\partial X_A} = 0.3 X_A^{-0.7} Y_A^{0.7} - \frac{1}{2} \lambda = 0$$

$$\frac{\partial U_A}{\partial Y_A} = 0.7 X_A^{0.3} Y_A^{-0.3} - \frac{1}{3} \lambda = 0$$

$$\frac{0.3 X_A^{-0.7} Y_A^{0.7}}{0.7 X_A^{0.3} Y_A^{-0.3}} = \frac{\frac{1}{2}}{\frac{1}{3}}$$

$$\frac{0.3 Y_A}{0.7 X_A} = \frac{3}{2}$$

$$0.6 Y_A = 0.21 X_A$$

$$0.2 Y_A = 0.7 X_A$$

$$\boxed{Y_A = \frac{0.7}{0.2} X_A}$$

$$\frac{1}{2} X_A + \frac{1}{3} Y_A = 10$$

$$\frac{1}{2} X_A + \frac{1}{3} \frac{0.7}{0.2} X_A = 10$$

$$\frac{X_A}{2} + \frac{0.7 X_A}{0.6} = 10$$

$$\frac{0.6 X_A + 1.4 X_A}{1.2} = 10$$

$$2 X_A = 12$$

$$\boxed{X_A = 6}$$

$$\frac{1}{2} (6) + \frac{1}{3} Y_A = 10$$

$$\frac{1}{3} Y_A = 7$$

$$\boxed{Y_A = 21}$$

Utility maximization by B:

$$\begin{aligned} \text{Maximize} \quad & U_B = X_B^{0.5} Y_B^{0.5} \\ \text{Subject to} \quad & \frac{1}{2} X_B + \frac{1}{3} Y_B = 1(10) \end{aligned}$$

$$\frac{\partial U_B}{\partial X_B} = 0.5 X_B^{-0.5} Y_B^{0.5} - \frac{1}{2} \lambda = 0$$

$$\frac{\partial U_B}{\partial Y_B} = 0.5 X_B^{0.5} Y_B^{-0.5} - \frac{1}{3} \lambda = 0$$

$$\frac{0.5 X_B^{-0.5} Y_B^{0.5}}{0.5 X_B^{0.5} Y_B^{-0.5}} = \frac{\frac{1}{2}}{\frac{1}{3}}$$

$$\frac{Y_B}{X_B} = \frac{3}{2}$$

$$3X_B = 2Y_B$$

$$\boxed{X_B = \frac{2}{3} Y_B}$$

$$\frac{1}{2} \left(\frac{2}{3} Y_B \right) + \frac{1}{3} Y_B = 10$$

$$\frac{1}{3} Y_B + \frac{1}{3} Y_B = 10$$

$$2Y_B = 30$$

$$\boxed{Y_B = 15}$$

$$\frac{1}{2} X_B + \frac{1}{3} (15) = 10$$

$$\frac{1}{2} X_B = 5$$

$$\boxed{X_B = 10}$$

c)

Market clearing requires:

$$(i) \quad X_A^* + X_B^* = X^* = 2L_x^*$$

$$(ii) \quad Y_A^* + Y_B^* = Y^* = 3L_y^*$$

$$(iii) \quad L_x^* + L_y^* = L^A + L^B = 20$$

$$(i) \text{ gives us } 6 + 10 = 2L_x^* \quad \Rightarrow \quad L_x^* = 8$$

$$(ii) \text{ gives us } 21 + 15 = 3L_y^* \quad \Rightarrow \quad L_y^* = 12$$

$$(iii) \text{ is satisfied } \Rightarrow \quad L_x^* + L_y^* = L^A + L^B = 20 \quad \Rightarrow \quad 8 + 12 = 10 + 10 = 20$$

3. Suppose there are only two individuals in a society. The demand curve for mosquito control for person A is given by

$$q_A = 100 - P.$$

For person B, the demand curve for mosquito control is given by

$$q_B = 200 - P.$$

a) Suppose mosquito control is a pure public good. What would be the optimal level of this activity if it could be produced at a constant marginal cost of TL120 per unit? Also, draw a diagram to indicate the optimal level of this activity.

b) If mosquito control were left to the private market, how much might be produced?

c) If the government were to produce the optimal amount of mosquito control, how much will this cost? How should the tax bill for this amount be allocated between the individuals if they are to share it in proportion to benefits received from mosquito control?

a) To find the total demand for mosquito control, demand curves must be summed vertically. Letting Q be the total quantity of mosquito control, the individuals' marginal valuations are:

$$P_A = 100 - Q$$

$$P_B = 200 - Q$$

$$\text{Total Valuation} = P_A + P_B = 300 - 2Q$$

Equating this total valuation (or $\sum MB$) to $MC = 120$ yields

$$300 - 2Q = 120$$

$$Q = 90$$

b) In the private market, $P = MC = 120$

At this price, A will demand zero units, and B will demand 80 units. Therefore, the output will be less than optimal.

$$Q = 100 - P_A$$

$$c) \quad 90 = 100 - P_A$$

$$P_A = 10$$

$$Q = 200 - P_B$$

$$90 = 200 - P_B$$

$$P_B = 110$$

A tax price of TL10 for A and TL110 for B will result in each individual demanding $Q=90$ and tax collections will exactly cover the per unit cost of mosquito control.

$$TC = (90)(120) = 10800$$

$$\text{Share of A} = (10)(90) = 900$$

$$\text{Share of B} = (110)(90) = 9900$$

$$900 + 9900 = 10800$$

4. There are 200 kilograms of food that must be allocated between two individuals on an island. The utility function of the first individual is given by

$$U_1 = \sqrt{F_1}$$

where F_1 is the quantity of food consumed by the first individual. For the second individual, utility is given by

$$U_2 = \frac{1}{2}\sqrt{F_2}$$

where F_2 is the quantity of food consumed by the second individual.

- If the food is allocated equally between the individuals, how much utility will each receive?
- How should food be allocated between the individuals to assure equality of utility?
- How should food be allocated so as to maximize the sum of two individuals' utilities?
- Suppose that both individuals agree on a social welfare function of the form

$$W = U_1^{1/2} U_2^{1/2}$$

How should food be allocated between two individuals so as to maximize social welfare?

$$U_1 = \sqrt{100} = 10$$

$$\text{a) } U_2 = \frac{1}{2}\sqrt{100} = 5$$

$$U_1 = U_2$$

$$\sqrt{F_1} = \frac{1}{2}\sqrt{F_2}$$

$$\text{b) } (\sqrt{200 - F_2})^2 = \left(\frac{1}{2}\sqrt{F_2}\right)^2$$

$$F_2 = 160$$

$$F_1 = 40$$

$$MU_1 = MU_2$$

$$\frac{1}{2\sqrt{F_1}} = \frac{1}{4\sqrt{F_2}}$$

$$\text{c) } \left(\frac{1}{2\sqrt{200 - F_2}}\right)^2 = \left(\frac{1}{4\sqrt{F_2}}\right)^2$$

$$\frac{1}{4(200 - F_2)} = \frac{1}{16F_2}$$

$$F_2 = 40$$

$$F_1 = 160$$

d)

$$\text{Maximize } U_1^{1/2} U_2^{1/2} + \lambda[200 - F_1 - F_2]$$

$$F_1^{1/4} \left(\frac{1}{2} \right)^{1/2} F_2^{1/4} + \lambda[200 - F_1 - F_2]$$

$$\frac{\frac{\partial U_2}{\partial F_2}}{\frac{\partial U_1}{\partial F_1}} = \frac{\frac{\partial W}{\partial U_1}}{\frac{\partial W}{\partial U_2}}$$

$$\frac{\frac{1}{4\sqrt{F_2}}}{\frac{1}{2\sqrt{F_1}}} = \frac{\frac{1}{2} \frac{U_2^{1/2}}{U_1^{1/2}}}{\frac{1}{2} \frac{U_1^{1/2}}{U_2^{1/2}}}$$

$$\frac{\frac{1}{4\sqrt{F_2}}}{\frac{1}{2\sqrt{F_1}}} = \frac{U_2}{U_1}$$

$$\frac{\frac{1}{4\sqrt{F_2}}}{\frac{1}{2\sqrt{F_1}}} = \frac{\frac{1}{2}\sqrt{F_2}}{\sqrt{F_1}}$$

$$F_2 = F_1 = 100$$

5. The music industry has characteristics of a public good. For example, songs played on the radio or television can be enjoyed without the consumer paying to buy a tape or a CD. However, the consumer might indirectly pay by buying the products of the firm that runs the commercials.

a) Even considering the indirect payment through advertised products explain how the music industry can have a free rider problem.

b) Given that the music industry has numerous free riders and that the music industry is profitable, how do you think the price of a tape/CD compares to its opportunity cost of production?

a) A free rider can exist in the music industry if someone listens to the music but buys neither the tapes/CDs nor the products advertised on the radio or TV.

b) If the music industry is profitable, then the price of a tape/CD is equal to or greater than its opportunity cost of production. Whether or not the music industry has free riders has no bearing on whether the price is more or less than production cost. What the free rider problem does mean is that, a misallocation of resources exists. A handful of people are paying to have tapes/CDs produced, while others benefiting from the music do not pay. Thus, the price is lower and the quantity produced is less than it needs to achieve an optimal allocation.

6. Suppose that the production possibility frontier for an economy that produces one public good (G) and one private good (X) is given by

$$X^2 + 100G^2 = 5000$$

This economy is populated by 100 identical individuals, each with a utility function of the form

$$U = \sqrt{X_i G}$$

where X_i is the individual's share of private good production ($X_i = X/100$).

a) What are the optimal production levels for G and X?

b) What would the typical individual's utility level be?

a) $X^2 + 100G^2 = 5000$ is the production possibility frontier's (PPF) equation.

Taking the total derivative of PPF:

$$2XdX + 200GdG = 0$$

$$MRT = -\frac{dX}{dG} = \frac{200G}{2X} = \frac{100G}{X}$$

$$\text{Individual } MRS_i = \frac{MU_G}{MU_X} = \frac{X_i}{G} = \frac{X/100}{G}$$

For efficiency:

$$\sum MRS_i = MRT$$

$$\frac{X}{G} = \frac{100G}{X}$$

$$X = 10G$$

Using PPF:

$$100G^2 + 100G^2 = 5000$$

$$G = 5$$

$$X = 50$$

b)

$$U = \sqrt{X_i G}$$

$$X_i = \frac{X}{100} = \frac{50}{100} = 0.5$$

$$U_i = \sqrt{(0.5)(5)} = \sqrt{2.5} = 1.58$$

7. The demand schedule for education in Brightland is as follows:

Number of Students (thousands)	Tuition fee (dollars per year)
0	5000
100	4000
200	3000
300	2000
400	1000
500	500
600	250

The marginal cost of providing education is \$3000 per year. There are external benefits of \$2000 per student.

a) If the market for education in Brightland is private with no government intervention, how many students enroll in school and what is the tuition fee?

b) If the government wants to achieve allocative efficiency, how many student school places does it make available, and how big is the subsidy per student?

a) The tuition fee and the number of students enrolling are determined by the intersection of the private supply and demand curves. Supply curve is perfectly elastic at \$3000 per year. At this tuition 200,000 students enroll.

b) Allocative efficiency is attained when output is such that marginal social cost equals marginal social benefit. In the case at hand the marginal social cost = marginal private cost which is \$3000. The key point is that the marginal social benefits are \$2000 higher than marginal private benefits. Hence, allocative efficiency requires that 400,000 students be enrolled. At this level of enrollment the marginal private benefits are \$1000. Thus it is necessary to add a subsidy of \$2000 (which, of course, equals marginal external benefits) to the marginal private benefit in order to induce enrollment of 400,000 students.

8. The demand for oil which is produced in an oil company's refinery is

$$Q = 50 - P.$$

The cost function of the refinery is given as

$$C(Q) = 8 + 4Q.$$

It is also known that there is a constant marginal external cost of \$6 per unit of oil production resulting from environmental damage associated with production.

- a) As a profit maximizer how much would this company like to produce?
- b) Is the amount you find in part (a) a socially desirable level of output? Explain.
- c) If your answer to (b) is no, what is the socially desirable level of output?
- d) If the government decides to use corrective tax to force this oil company to produce the socially desirable level of output, what will be the total amount of tax the company has to pay?

a) Profit maximizing condition: $MR = MC$.

$$MR = 50 - 2Q$$

$$MC = 4$$

$$50 - 2Q = 4$$

$$46 = 2Q$$

$$Q = 23$$

b) No, it is not. The oil refinery in maximizing its profits does not take into consideration the negative externality.

c) We have to include the environmental damage.

$$MR = 50 - 2Q$$

$$MC' = 4 + 6 = 10$$

$$MR = MC'$$

$$50 - 2Q = 10$$

$$Q = 20$$

d) To produce the socially desirable level of output which is 20 units, a tax of \$6 per unit of output should be imposed. Total tax collected: $20 \times 6 = 120$

$$X^2 + Y^2 = 50$$

9. Suppose $U_i(X_i, Y_i) = X_i Y_i \quad i = 1, 2$

$$W(U_1, U_2) = U_1 U_2$$

How much of x and y should be produced and how should it be distributed?

Maximize $W(U_1, U_2) = U_1 U_2$

$$U_1 = X_1 Y_1$$

$$U_2 = X_2 Y_2$$

Subject to $X^2 + Y^2 = 50$

$$X_1 + X_2 = X$$

$$Y_1 + Y_2 = Y$$

Maximize $U_1 U_2 + \lambda [X^2 + Y^2 - 50]$

Subject to $X = X_1 + X_2$

$$Y = Y_1 + Y_2$$

Maximize $X_1 Y_1 X_2 Y_2 + \lambda [X^2 + Y^2 - 50]$

Subject to $X = X_1 + X_2$

$$Y = Y_1 + Y_2$$

Social optimum requires:

$$\frac{\frac{\partial U_2}{\partial X_2}}{\frac{\partial U_1}{\partial X_1}} = \frac{\frac{\partial W}{\partial U_1}}{\frac{\partial W}{\partial U_2}} \Rightarrow \frac{Y_2}{Y_1} = \frac{U_2}{U_1} = \frac{Y_2 X_2}{Y_1 X_1} \Rightarrow X_2 = X_1 \quad \text{and} \quad Y_2 = Y_1$$

Hence,

$$\text{Max} \quad X_1^2 Y_1^2 + \lambda [(2X_1)^2 + (2Y_1)^2 - 50]$$

$$\frac{\partial \ell}{\partial X_1} = 2X_1 Y_1^2 + 8X_1 \lambda = 0$$

$$\frac{\partial \ell}{\partial Y_1} = 2X_1^2 Y_1 + 8Y_1 \lambda = 0 \quad \Rightarrow \quad \frac{2X_1 Y_1^2}{2X_1^2 Y_1} = \frac{-8X_1 \lambda}{-8Y_1 \lambda} \quad \Rightarrow \quad \frac{Y_1}{X_1} = \frac{X_1}{Y_1} \quad \Rightarrow \quad \boxed{X_1 = Y_1}$$

$$\frac{\partial \ell}{\partial \lambda} = 4X_1^2 + 4Y_1^2 - 50 = 0$$

$$4X_1^2 + 4Y_1^2 = 50 \quad \Rightarrow \quad 8X_1^2 = 50 \quad \Rightarrow \quad X_1^2 = \frac{50}{8} \quad \Rightarrow \quad X_1 = \frac{5}{2}$$

$$\text{Therefore,} \quad \boxed{X_1 = X_2 = Y_1 = Y_2 = \frac{5}{2}} \quad \text{and} \quad \boxed{X = Y = 5}$$

10. Compare and contrast Pareto optimality conditions for a two goods (X and Y), two inputs (K and L) and two individuals (A and B) economy for the cases where both goods are private and where only one of the goods is private and the other is public.

CASE 1: Both goods are private

Conditions for Pareto optimality are:

Efficient production: $MRTS_{KL}^X = MRTS_{KL}^Y$

Efficient Consumption: $MRS_{XY}^A = MRS_{XY}^B$

Efficient Product-Mix: $MRS_{XY}^A = MRT_{XY} = MRS_{XY}^B$

CASE 2: One good is public (X) and one good is private (Y)

Except the efficient production condition the rules for Pareto optimality change when one of the goods is treated as a public good. Therefore,

Efficient production: $MRTS_{KL}^X = MRTS_{KL}^Y$ does not change. It stays the same.

However the other two rules do not hold any more. Instead we have the following condition to hold when one of two goods is a public good.

$$MRS_{XY}^A + MRS_{XY}^B = MRT_{XY}$$

11. Alkent Apartments has 100 residents who are concerned about security. The table below gives the total cost of hiring a 24-hour security guard service as well as the marginal benefit to each of the residents.

- Why is a security guard a public good for the residents of Alkent Apartments?
- If each of the residents acts individually, how many guards will be hired?
- Complete the last column of the table by computing the marginal benefit of security guards to the apartments; i.e., to all residents together.

Number of guards	Total cost per day (NTLs)	Marginal benefit per resident (NTLs)	Marginal benefit to all residents (NTLs)
1	300	10	
2	600	4	
3	900	2	
4	1200	1	

Now suppose that the residents form an Apartment Council which acts as a governing body in order to address the security issue.

- What is the optimal (electively efficient) number of guards? What is the net benefit at the optimal number of guards?
- Show that net benefits is less for either one less guard or for one more guard than the net benefit for the optimal number of guards.

a) A security guard is a public good because in this case it has the features of nonrivalry and nonexcludability.

b) If each resident must act individually in hiring a security guard none will be hired since each resident receives only 10 NTL in benefit from the first guard which costs 300 NTL per day.

c)

Number of guards	Total cost per day (NTLs)	Marginal benefit per resident (NTLs)	Marginal benefit to all residents (NTLs)
1	300	10	1000
2	600	4	400
3	900	2	200
4	1200	1	100

d) If the apartment council hires each guard for whom the $MB > MC$, then they will hire the optimal number of guards. The MC of each additional guard is 300 NTL. The total MB of first guard is 1000, so she will be hired. Similarly, the second guard will also be hired. But for the third guard, $MB < MC$, so she will not be hired. Therefore, the optimal number of guards is 2. The net benefit of hiring 2 guards is:

Total benefit – Total cost = Net benefit
 1400 - 600 = 800

e) For 1 guard, net benefit is:

Total benefit – Total cost = Net benefit
 1000 - 300 = 700

For 3 guards, net benefit is:

Total benefit – Total cost = Net benefit
 1600 - 900 = 700

12. Suppose there are only two people, Thelma and Louise, who must split a fixed income of 200 TL. For Thelma, the marginal utility of income is

$$MU_T = 500 - 2Y_T$$

while for Louise, marginal utility is

$$MU_L = 500 - 6Y_L$$

where Y_T , Y_L are the amounts of income to Thelma and Louise, respectively.

- What is the optimal distribution of income?
- What is the optimal distribution if society values only the utility of Thelma?
- What is the optimal distribution of income if $MU_T = MU_L = 500$?

$$MU_T = MU_L$$

$$500 - 2Y_T = 500 - 6Y_L$$

$$Y_T + Y_L = 200$$

$$Y_T = 200 - Y_L$$

$$\text{a) } 500 - 2(200 - Y_L) = 500 - 6Y_L$$

$$8Y_L = 400$$

$$Y_L = 50$$

$$Y_T = 150$$

- If only Thelma matters, then give money to Thelma until $MU_T = 0$. So,

$$500 - 2Y_T = 0$$

If total amount of money is 200 TL, hence giving all the money to Thelma is optimal. In fact we would like to give her up to 250 TL because it is the amount that makes $MU_T = 0$.

- $MU_T = MU_L$ for all levels of income. Hence, society is indifferent among all distributions of income.

13. The market demand function for cordless phones is given as:

$$Q = 600 - 4P$$

The industry marginal cost function is:

$$MC = 0.5Q$$

- a) By assuming perfect competition, find the equilibrium price and quantity for this market if the government imposes a unit tax of TL15. How much of the tax is paid by the consumers and the producers? (Note: impose the tax on the seller.)
- b) Again, by assuming perfect competition, find the equilibrium price and quantity for this market if the government imposes an ad valorem tax of 50%. How much of the tax is paid by the consumers and the producers? (Note: impose the tax on the buyer.)
- c) By assuming that the industry is controlled by a monopolist, find the equilibrium price and quantity for this market if the government imposes a unit tax of TL15. How much of the tax is paid by the consumers and the monopolist? (Note: impose the tax on the seller.)
- d) Find out the ad valorem tax rate which gives the same price and output combination as in part (c). (Note: impose the tax on the buyer.)

a) $MC = 0.5Q$ $Q = 2P$

$$600 - 4P = 2P$$

$$P = 100 \quad \text{and} \quad Q = 200 \quad (\text{equilibrium } P \text{ and } Q \text{ before the tax})$$

$$P_D = P_S + t$$

$$P_D = 150 - \frac{Q}{4}$$

$$P_S = 0.5Q$$

$$150 - \frac{Q}{4} = 0.5Q + 15$$

$$Q = 180$$

$$P_D = 150 - \frac{180}{4} = 105$$

$$P_S = 0.5(180) = 90$$

$$P_D - P_S = t$$

$$105 - 90 = 15$$

$$\text{Consumers pay : } 105 - 100 = 5$$

$$5(180) = 900$$

$$\text{Producers pay : } 100 - 90 = 10$$

$$10(180) = 1800$$

$$\text{Total tax payment : } 900 + 1800 = 2700$$

b) Initially before the tax $P = 100$ and $Q = 200$

$$(1 - \tau)P_D = P_S$$

$$(1 - \tau)(150 - \frac{Q}{4}) = P_S$$

$$(1 - 0.5)(150 - \frac{Q}{4}) = 0.5Q$$

$$Q = 120$$

$$P_D = 150 - \frac{120}{4} = 120$$

$$P_S = 0.5(120) = 60$$

$$\text{Consumers pay : } 120 - 100 = 20$$

$$20(120) = 2400$$

$$\text{Producers pay : } 100 - 60 = 40$$

$$40(120) = 4800$$

$$\text{Total tax payment : } 2400 + 4800 = 7200$$

c) Initially (no tax case):

$$P = 150 - 0.25Q$$

$$TR = PQ = (150 - 0.25Q)(Q) = 150Q - 0.25Q^2$$

$$MR = 150 - 0.5Q$$

$$MC = 0.5Q$$

$$\text{At equilibrium } MR = MC$$

$$150 - 0.5Q = 0.5Q$$

$$Q = 150 \text{ and } P = 112.5$$

After the imposition of the tax:

$$MC' = 0.5Q + 15$$

$$MR = 150 - 0.5Q$$

$$MR = MC'$$

$$150 - 0.5Q = 0.5Q + 15$$

$$Q = 135$$

$$P_D = 150 - 0.25(135) = 116.25$$

$$P_S = P_D - t$$

$$101.25 = 116.25 - 15$$

Consumers pay : $116.25 - 112.5 = 3.75$
 $3.75 (135) = 506.25$
Producers pay : $112.5 - 101.25 = 11.25$
 $11.25 (135) = 1518.75$
Total tax payment : $506.25 + 1518.75 = 2025$

d)
 $(150 - 0.5Q)(1 - \tau) = 0.5Q$
 $(150 - 0.5(135))(1 - \tau) = 0.5(135)$
 $82.5(1 - \tau) = 67.5$
 $15 = 82.5\tau$
 $\tau = 0.18$

14. Consider the following two cases: Case I and Case II. In each case there are three voters (A, B, and C) to vote for three alternative levels of budget activity 1 (low), 2 (medium) and 3 (high). The results of an election between any pair of alternatives will be determined by simple majority vote. The voter rankings are given in the table below for Case I and Case II.

	CASE I			CASE II		
	A	B	C	A	B	C
First Choice	1	2	3	3	1	2
Second Choice	2	3	1	2	2	1
Third Choice	3	1	2	1	3	3

- Indicate for each case whether there is a Condorcet winner or not. Give brief explanation.
- Indicate for each case whether the preferences of all voters are single-peaked or not. Explain and draw diagrams for Case I and II separately by placing the levels of budget on the horizontal axis and levels of choices on the vertical axis.
- Consider the following case: Case III. Similar to Cases I and II, rankings between different levels of budget (low, medium and high) are also stated for Case III. In addition to that, the voters A, B, and C are given 10 voting points to allocate between three alternative budgets. Assume that the voter records her/his true preferences.

	CASE III					
	A		B		C	
	Rank	Points	Rank	Points	Rank	Points
Low Budget	1	6	3	1	3	1
Medium Budget	2	3	1	5	2	2
High Budget	3	1	2	4	1	7

State clearly which alternative will win under majority voting, plurality voting and point voting. Give brief explanation.

a) To decide on this issue we must make pairwise comparison between different alternatives.

For Case I:

Between 1 (low) and 2 (medium); 1 (low) is preferred.

Between 1 (low) and 3 (high); 3 (high) is preferred.

Between 2 (medium) and 3 (high); 2 is preferred.

Each time a different alternative wins. There is no Condorcet winner. Instead there is cycling of votes (voting paradox).

For Case II:

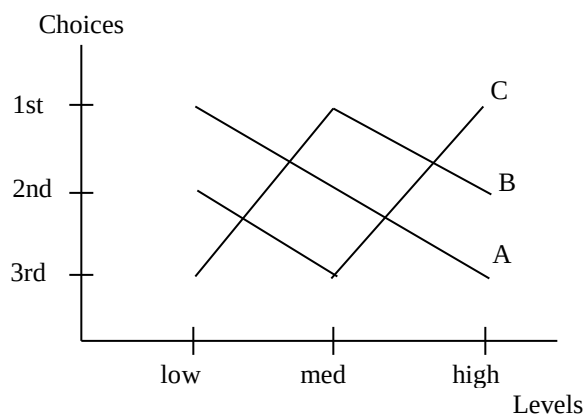
Between 1 (low) and 2 (medium); 2 (medium) is preferred.

Between 1 (low) and 3 (high); 1 (low) is preferred.

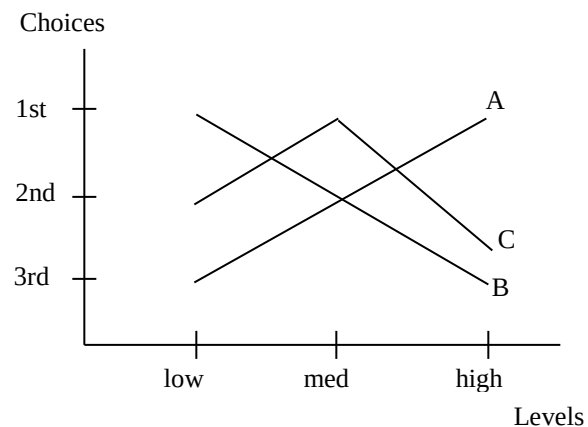
Between 2 (medium) and 3 (high); 2 (medium) is preferred.

There is a Condorcet winner: it is 2 (medium budget).

b)



Case I



Case II

For Case I:

Voters A and B have single peaked preferences but voter C does not.

For Case II:

All voters have single peaked preferences. Median voter's (voter C) most preferred outcome (medium budget) will be the outcome chosen.

c) For majority voting: Using ranks we make pairwise comparison.

Between L and M; M is preferred.

Between L and H; H is preferred.

Between M and H; M is preferred.

Therefore, as a result M (medium budget) wins.

For plurality voting: Adding ranks, we choose the alternative with the lowest rank totals.

$$L = 1 + 3 + 3 = 7$$

$$M = 2 + 1 + 2 = 5$$

$$H = 3 + 2 + 1 = 6$$

Therefore, as a result M (medium budget) wins.

For point voting: Adding points, we choose the alternative with the highest points.

$$L = 6 + 1 + 1 = 8$$

$$M = 3 + 5 + 2 = 10$$

$$H = 1 + 4 + 7 = 12$$

Therefore, as a result H (high budget) wins.

15. The following table gives information about various tax brackets and corresponding marginal tax rates (MTR) for a country.

<u>Tax Brackets</u>	<u>MTR</u>	<u>ATR (bob)</u>	<u>ATR (eob)</u>
0-5000	25%		
5000-10000	30%		
10000-20000	35%		
20000-40000	40%		
40000-80000	45%		
80000-160000	50%		
160000+	55%		

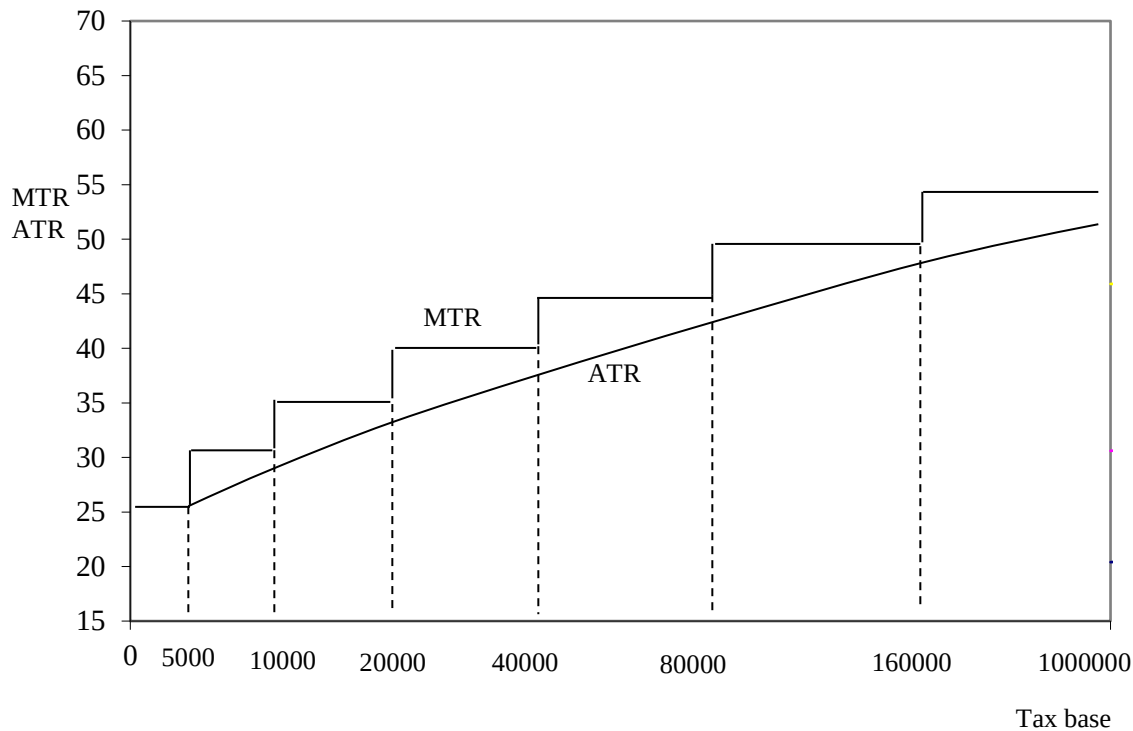
a) Calculate ATR for the beginning of bracket (bob) and end of bracket (eob). Show your calculations. For the last bracket take the upper income level as one million dollars.

b) Draw a graph which shows the relationship between ATR and MTR.

c) Is this taxation system progressive? Explain.

<u>a) Tax Brackets</u>	<u>MTR</u>	<u>ATR (bob)</u>	<u>ATR (eob)</u>
0-5000	25%	0	25
5000-10000	30%	25	27.5
10000-20000	35%	27.5	31.25
20000-40000	40%	31.25	35.625
40000-80000	45%	35.625	40.3125
80000-160000	50%	40.3125	45.15625
160000+	55%	45.15625	53.42

b)



c) This taxation system is progressive as ATR is eventually increasing and $ATR < MTR$.

16. Given that $I_0=10000$, $I_1=30000$, $T_0=1400$ and $T_1=5400$, where I_0 and I_1 denote income levels and T_0 and T_1 denote tax liabilities at I_0 and I_1 , respectively.

a) Calculate average rate progression coefficient. Interpret the coefficient.

b) Calculate liability progression coefficient. Interpret the coefficient.

c) Consider the case that tax liabilities are to be increased by 10 percent of the amount of the taxes currently paid. Does this new case make the tax system more progressive? Show your calculations.

$$a) \frac{\left(\frac{T_1}{I_1} - \frac{T_0}{I_0} \right)}{I_1 - I_0} = \frac{\left(\frac{5400}{30000} - \frac{1400}{10000} \right)}{30000 - 10000} = \frac{0.18 - 0.14}{20000} = 0.000002$$

There is progression but it is very small (as the coefficient is greater than zero).

$$b) \frac{\frac{T_1 - T_0}{T_0}}{\frac{I_1 - I_0}{I_0}} = \frac{\frac{5400 - 1400}{1400}}{\frac{30000 - 10000}{10000}} = \frac{2.85714}{2} = 1.42857$$

There is progression.

$$c) \frac{\left(\frac{5400 (1.1)}{30000} - \frac{1400 (1.1)}{10000} \right)}{30000 - 10000} = \frac{0.198 - 0.154}{20000} = 0.0000022$$

Progressivity increases when we use average rate progression coefficient.

$$\frac{\frac{5400 (1.1) - 1400 (1.1)}{1400 (1.1)}}{\frac{30000 - 10000}{10000}} = \frac{2.85714}{2} = 1.42857$$

No change in progressivity takes place when we use liability progression coefficient.

17. Suppose a community is considering two projects: reform welfare (RW) and stricter environmental standards (SES). The community has three groups of voters. The following table shows their benefits for each project. Note that a minus sign indicates a net loss.

	<u>WR</u>	<u>SES</u>
Group I	-1	-1
Group II	5	-2
Group III	-2	5

- Would the community as a whole be better off if each project were adopted? Explain briefly.
- What happens if the projects are voted on one at a time? Explain briefly.
- Which outcome will result with logrolling? Is it a desirable outcome? Explain briefly.

a) In order to answer this question, we have to find out the total net benefit for each project:

$$TNB_{WR} = (-1) + (5) + (-2) = 2$$

$$TNB_{SES} = (-1) + (-2) + (5) = 2$$

Total net benefits of each project are positive. Therefore, community as a whole would be better off if each project were adopted.

b) If the projects are voted one at a time:

for WR: Group I and Group III will say no, and Group II will say yes to the project. Therefore, the project fails 2 to 1.

for SES: Group I and Group II will say no, and Group III will say yes to the project. Therefore, the project fails 2 to 1.

Simply, no project would be implemented under these conditions.

c) Group II and Group III can make a deal. Group II can support the project SES, in return Group III can support the project WR. By this way both projects pass (2 to 1). Both parties as a result of this deal become better off. In this case Group II comes out ahead by $(5) + (-2) = 3$ with the logrolling. Also Group III comes out ahead by $(-2) + (5) = 3$ with the logrolling. Thus vote trading allows both of the projects to pass, which is a desirable outcome.

18. Suppose that you currently earn taxable income of TL100,000 per year. You are subject to a marginal tax rate of 50 percent. Currently, your average tax rate is 35 percent.

a) Calculate your annual tax payments.

b) Calculate the extra tax that you would pay per year if your annual income increased to TL110,000 and your average tax rate when your annual income is TL110,000.

a) $ATR = \text{tax payments} / \text{value of the tax base}$

$$\text{tax payments} / 100,000 = 0.35$$

$$\text{tax payments} = 0.35 \times 100,000 = 35,000$$

b) $MTR = \Delta \text{ in tax payments} / \Delta \text{ in value of the tax base}$

$$0.5 = (X - 35,000) / (110,000 - 100,000)$$

$X = 40,000$ = amount of tax paid when income is TL110,000

Therefore, the extra tax paid is TL5,000.

$$ATR = 40,000 / 110,000 = 0.36$$

19. Consider an income tax calculated from the formula

$$t_i = \phi(I_i - d)$$

where: t_i represents i 's annual tax bill,

$I_i \geq 0$ represents i 's annual income,

$d \geq 0$ represents the deductions,

$0 \leq \phi \leq 1$ is a parameter.

a) What is individual i 's marginal tax rate and average tax rate?

b) Are income taxes calculated from the above formula progressive? Explain your answer.

$$MTR = \frac{\partial t_i}{\partial I_i} = \phi$$

a)

$$ATR = \frac{t_i}{I_i} = \frac{\phi(I_i - d)}{I_i} = \phi - \frac{\phi d}{I_i}$$

$$b) \quad \frac{\partial ATR}{\partial I_i} = \frac{\phi d}{I_i^2}$$

The income taxes calculated from the above formula are progressive, because ATR increases with income.

20. A monopolist can produce at constant average and marginal cost of TL5 ($AC = MC = TL5$). The firm faces a market demand function given by $Q=53-P$.

a) Calculate the profit maximizing price and output level for the monopolist.

b) What output level would be produced by this industry if it were a perfectly competitive industry?

Also calculate the corresponding price.

Suppose the government imposes a per unit tax equal to TL1, which is to be collected from the firm.

c) Assuming that the industry is a monopoly industry, what would be the effects of this tax on price and quantity? What is the total amount of tax revenue raised by the government? How will the tax burden be distributed between consumers and the producer?

d) Assuming that the industry is a perfectly competitive industry, what would be the effects of this tax on price and quantity? What is the total amount of tax revenue raised by the government? How will the tax burden be distributed between consumers and the producers?

a) To maximize profits $MR = MC$

$$MR = 53 - 2Q$$

$$53 - 2Q = 5$$

$$Q = 24 \quad P = 29$$

b) In competitive market $P = MC$

$$53 - Q = 5$$

$$Q = 48 \quad P = 5$$

c) $MR = MC'$

$$53 - 2Q = 5 + 1$$

$$47 = 2Q$$

$$Q = 23.5$$

$$P_c = 53 - Q$$

$$= 53 - 23.5$$

$$P_c = 29.5$$

$$P_M = 29.5 - 1 = 28.5$$

Note: Remember that in class we considered exactly the same case (linear demand, horizontal MC curve) and find that if we have a unit tax then the after tax, price increases by exactly $\frac{1}{2}$ of the tax. That is what we have found here.

$$\text{Tax revenue: } 1 * 23.5 = 23.5$$

In the market the price has increased from 29 to 29.5 TL. Therefore consumers pay only $\frac{1}{2}$ of the tax and the other half is paid by the monopolist.

Tax burden is shared equally: $23.5 / 2 = 11.75$ TL.

d) $P = MC'$

$$53 - Q = 5 + 1$$

$$Q = 47$$

$$P_c = 6$$

$$P_p = 5$$

$$\text{Tax revenue: } 1 * 47 = 47$$

In the market the price has increased from 5 to 6 TL. Price has increased by the amount of the tax. Since MC is a horizontal line which shows a perfectly elastic supply curve, all of the tax (1 TL per unit and 47 TL as total amount of the tax) is paid by the consumers.